Appendix I

Lognormal and Delta Lognormal Distributions for Maximum Daily Loads

Draft

Fort Gibson Lake TMDL Report

Prepared for

Oklahoma Department of Environmental Quality

Water Quality Division

April 29, 2019 (Final)

February 9, 2022 (Revised)

By

Dynamic Solutions, LLC

Appendix I – Lognormal and Delta Lognormal Distributions for Maximum Daily Loads

Table of Contents

List of Tables

List of Figures

I-1 LOGNORMAL AND DELTA LOGNORMAL DISTRIBUTIONS FOR MAXIMUM DAILY LOADS

I-1.1 Introduction

Sources of nutrients and sediment loads to Fort Gibson Lake include the outflow from Lake Hudson as upstream boundary inflow to the Neosho River and upper Fort Gibson Lake, watershed runoff simulated with the HSPF model, and NPDES wastewater discharges from municipal and industrial facilities. The EFDC lake model was calibrated and validated to flow and water quality data collected during 2005-2006. Flow and loading data from 2006 were used to determine the overall load reduction of sediment, organic matter and nutrients required to attain compliance with water quality targets for Fort Gibson Lake.

Consistent with guidance from EPA (1991) for TMDL calculations, flow and pollutant loads from watershed runoff and NPDES wastewater dischargers can be represented as lognormal distributions by log transformation of the time series data sets. Log transformed flow and loading data of the upstream boundary inflow from Lake Hudson, however, exhibits considerable skewness at the low end of the flow and load distributions. The assumption that the lognormal distribution is a reasonable representation of the inflow from Lake Hudson as an external source of pollutant loads to Fort Gibson Lake is, therefore, not appropriate for the 2006 data set used to specify the upstream inflow boundary for the Fort Gibson Lake EFDC model.

Three documents available from EPA provide the statistical basis for determination of maximum daily loading (MDL) rates from annual loading rates. In 2007, EPA published "*Options for Expressing Daily Loads in TMDLs"* (EPA, 2007) in response to the "Anacostia Decision" (Grumbles, 2006). The statistical basis for the calculation of a daily loading rate from an annual average load was previously documented by EPA in technical guidance documents: "*Technical Guidance Manual for Performing Wasteload Allocations, Book VII: Permit Averaging Periods*" EPA (1984); and *"Technical Support Document for Water Quality-Based Toxics Control"* EPA (1991). These documents provide the assumptions and statistical methods for the equations, parameters, and calculation of maximum daily load limits based on long-term average loads and temporal variability of the pollutant load time series datasets. Appendix E of EPA (1991) and Section 2 of EPA (1984) present the rationale, equations and parameters based on the normal and lognormal distributions that can be used to calculate maximum daily loads. Appendix E of EPA (1991) also describes the delta lognormal distribution as a third statistical methodology for calculating maximum daily loads for data sets described by a mix of censored (i.e., less than detection limit) and non-censored above detection limit data sets.

I-1.2 Purpose and Objectives

Appendix I presents the methodology, equations and distribution parameters used to develop the TMDL calculations summarized in Section 5 of the Fort Gibson Lake TMDL report. Data and analyses presented in Appendix I meet the following three objectives:

Objective 1. The methodology, equations and parameters of the lognormal distribution are presented. As an example of the calculations, daily load data for TP are analyzed and presented to support the use of the lognormal distribution for watershed runoff and wastewater loading. Histograms and probability plots are presented to support application of the lognormal distribution for MDL calculations derived from watershed runoff and wastewater dischargers.

Objective 2. The methodology, equations and parameters of the delta lognormal distribution are presented. As an example, daily flow and TP loading data are analyzed and presented for the inflow from Lake Hudson to demonstrate, and support, application of the delta lognormal distribution as an appropriate representation of the time series of pollutant loads from the Lake Hudson inflow for the drought year conditions of 2006. Plots of the histogram and probability distribution for TP are presented to demonstrate that the delta lognormal distribution provides a robust statistical representation of pollutant loads exported from Lake Hudson for MDL calculations derived from the inflow from Lake Hudson.

Objective 3. The distribution parameters, existing long-term average loads, and calculations of the MDL's for the inflow from Lake Hudson, watershed runoff, and NPDES wastewater sources are summarized in tables. The TMDL and the load allocations (LA) and wasteload allocations (WLA) for TP, TN, TOC, and TSS, computed from the MDL's derived for the inflow from Lake Hudson, watershed runoff, and NPDES wastewater dischargers, are summarized in tables.

I-2 LOGNORMAL DISTRIBUTION FOR MAXIMUM DAILY LOADS

The equations used for calculating the maximum daily load from long-term average loads are based on the assumption that streamflow, water quality concentration and watershed stream loading data are log normally distributed. It has been well documented in numerous studies that a two-parameter lognormal distribution defined by the mean and variance of the log transformed data set provides a useful approximation to the probabilistic distribution of streamflow (Nash, 1994; Limbrunner et al., 2000; Vogel et al., 2005). Van Buren et al., (1997) and Di Toro (1984) also determined that water quality analyses based on an assumption of the lognormal probability distribution for both streamflow and water quality concentration are quite realistic for many streams and rivers, including waterbodies investigated in the United States.

I-2.1 Equations and Parameters of Lognormal Distribution

The time series for existing pollutant loading data are defined by a set of time series measurements of the random variable 'X' where 'y' is defined as the set of natural log transformation of the loading data:

$X =$ existing pollutant load data defined by time series

$y = \ln(X) =$ natural log transformation of pollutant load data, X

Equations for the log transformed data are presented for the lognormal distribution to define the lognormal parameters used to calculate the maximum daily loads for watershed runoff and wastewater dischargers. The existing mean or expected value (E_x), variance (V_x), and upper percentile limit (X_p) of the random variable distribution of 'X' expressed in the measured units of the original untransformed existing data set are given as follows:

$E(X) = \exp\,(\mu + 0.5\sigma^2)$-- Equation 1

$V(X) = \exp(2\mu + \sigma^2)\,[\exp(\sigma^2) - 1]$ -- Equation 2

$X_p = \exp(\mu + Z\sigma)]$-- Equation 3

Estimates of these parameters are derived from substitution of the values for the sample mean (μ) and sample variance (σ^2) calculated from the natural log transformed observed data set [y=ln (X)] where:

μ = mean of log transformed data set, y

σ^2=variance of log transformed data set, y

σ = standard deviation of log transformed data set, y

z_p = p^th percentile of the 1-sided standard normal distribution (z_p=1.645 for 95^th percentile and z_p=2.326 for 99^th percentile)

The daily maximum variability factor (VF_p) for the p^{th} percentile of the standard normal distribution, based on the ratio of the high value to average value of a lognormal distribution, is defined by Kahn and Rubin (1989) and EPA (1991) in Appendix E as:

$VF_p = \frac{X_p}{E_x}$ -- Equation 4

$VF_p = \exp\,(z_p\sigma - 0.5\sigma^2)$ -- Equation 5

In EPA (1991), the variability factor (VF_p) is defined as the ratio of X_p (the upper limit of the confidence interval of the log transformed data) and the expected mean value E_x of the log transformed loading data. As shown in Table 5-2 of EPA (1991), the equation for the variability factor ratio is used with the coefficient of variation (CV) to compute the table of Long-Term Average (LTA) "LTA multipliers" to derive the maximum daily load (MDL) based on the reduced LTA needed to meet water quality targets.

The coefficient of variation (CV_x) for the untransformed load data set is computed from the log transformed parameter value for variance (σ^2):

$CV_x = [\exp(\sigma^2) - 1]^{1/2}$ -- Equation 6

The variability factor is then used to calculate the MDL from the reduced LTA needed to attain compliance with water quality targets as follows:

$MDL = LTA * VF_p = LTA * LTA\ Multiplier$ --- Equation 7

Substitution of the exponential term for VF_p from above yields the lognormal distribution equation for the MDL given in EPA's *Options for Expressing Daily Loads in TMDLs* (EPA, 2007):

$MDL = LTA\,\exp\,(z_p\sigma - 0.5\sigma^2)$ --- Equation 8

The long-term average reduced load (LTA) is computed from the existing long-term average load [E(X)] computed from the log transformation of the observed load data and the lake model-derived percent reduction (%R) expected to meet water quality targets in the lake.

$LTA = E(X)(1 - \%R)$ --- Equation 9

I-2.2 Lognormal Representation of Watershed Runoff and Wastewater Loads

Although it is well documented in the literature, data are presented to show that the assumption of a lognormal distribution for watershed runoff and NPDES wastewater loading data holds true for the TMDL analysis for Fort Gibson Lake. Total Phosphorus (TP) loading data derived from HSPF-modeled watershed runoff and NPDES wastewater sources is used as an example data set to show that the lognormal distributions for watershed runoff and wastewater loading data are appropriate assumptions for MDL calculations for Fort Gibson Lake.

As shown by the approximations to a bell shaped curve of the histogram (Figure 1) and a linear relationship of the probability plot (Figure 2), the log transformed TP load data for watershed runoff shows a good approximation to a lognormal distribution. Similarly, the histograms and probability plots for TP loading from large (Figure 3 and Figure 4) and small (Figure 5 and Figure 6) NPDES wastewater sources also show reasonable approximations for a lognormal distribution. Probability plots of log transformed TP load data for watershed runoff (r^2=0.996), large (r^2=0.95) and small wastewater loads (r^2=0.98) demonstrate good approximations of the linear relationship with the Z-score statistic; this confirms the assumption of a lognormal distribution for these pollutant sources. As flow is common to all loads derived from wastewater and watershed runoff, Total Nitrogen (TN), Total Organic Carbon (TOC), and Total Suspended Solids (TSS) loads also display similar lognormal distributions.

An example of the MDL calculations derived from the lognormal equations and parameters are presented for Total Phosphorus for watershed runoff, small and large wastewater. Wastewater facilities were split out as “small” (< 1 MGD) and “large” (> 1 MGD) so that the lognormal distribution would provide reasonable statistical representations of the loading data. Three “large” facilities, accounting for 92% of the total effluent flow of 9.0 MGD, were defined by effluent flows of 3.5 MGD (39%), 3.4 MGD (38%), and 1.4 MGD (15%) of the effluent flow. The three “small” facilities, accounting for 8% of the 9.0 MGD total effluent flow, were defined by effluent flows of 0.36 MGD (4.1%), 0.2 MGD (2.2%) and 0.14 MGD (1.6%). Similar MDL calculations derived from the delta lognormal distribution equations and parameters are presented for TP for the inflow from Lake Hudson in Table 4.

The LTA for TP for Fort Gibson Lake is derived as the sum of the reduced LTAs calculated for each source and the TMDL for TP for Fort Gibson Lake is derived as the sum of the MDLs calculated for each source as follows:

$$TMDL = MDL\ (Inflow) + MDL\ (Watershed) +\ MDL\ (Small\ WWTP) +\ MDL\ (Large\ WWTP)$$

-- Equation 10

Table 1, Table 2 and Table 3 present the statistics derived for the lognormal loading analysis of Total Phosphorus for watershed runoff and small and large wastewater facilities.

Table 1 - Lognormal Parameters and Estimation of Maximum Daily Load for Total Phosphorus for Watershed (HSPF) Runoff in 2006

Watershed (HSPF) Runoff to Lake Ft. Gibson: 2006 Lognormal Distribution Total-Phosphorus			
Watershed Parameters, Log Transformed			
Ln (X, kg/day)		Arithmetic (X, kg/day)	
N =	364		
μ =	3.782921	E(X)=	187.45
σ^2 =	2.901183	V(X)=	604212.2
σ =	1.703286	s(X)=	777.3109
CV =	0.450257	CV(X)=	4.146764
Min=	-0.10031	Min(X)=	0.904556
Max=	8.612423	Max(X)=	5499.557
		1-sided, α=	0.05
		p (1-α)=	0.95
		Arg (Ø)=	0.95
		r^2=	0.9958
		z(p)=	1.645
Maximum Daily Load Parameters			
Upper Limit		X(p)=	724.025
Variability Factor		VF(p)=X(p)/E(X)=	3.862497
Existing Average		E(x)=	187.45
% Removal		%R=	45%
Long Term Average		LTA= E(X*)*(1-%R)=	103.0975
Max Daily Load		MDL= LTA*VF(p)=	398.2138

Table 2 - Lognormal Parameters and Estimation of Maximum Daily Load for Total Phosphorus for Small Wastewater Loads in 2006

Small Wastewater to Lake Ft. Gibson: 2006 Lognormal Distribution Total-Phosphorus			
Small Wastewater Parameters, Log Transformed			
Ln(X, kg/day)		Arithmetic (X, kg/day)	
N =	364		
μ =	1.341887	E(X)=	4.04809
σ^2 =	0.112716	V(X)=	1.955206
σ =	0.335732	s(X)=	1.398287
CV =	0.250194	CV(X)=	0.345419
Min=	0.656034	Min(X)=	1.927135
Max=	2.089887	Max(X)=	8.083999
		1-sided, α=	0.05
		p (1-α)=	0.95
		Arg (Ø)=	0.95
		r^2=	0.98
		z(p)=	1.645
Maximum Daily Load Parameters			
Upper Limit		X(p)=	6.647008
Variability Factor		VF(p)=X(p)/E(X)=	1.642011
Existing Average		E(x)=	4.04809
% Removal		%R=	45%
Long Term Average		LTA= E(X*)*(1-%R)=	2.22645
Max Daily Load		MDL= LTA*VF(p)=	3.655854

Table 3 - Lognormal Parameters and Estimation of Maximum Daily Load for Total Phosphorus for Large Wastewater Loads in 2006

Large Wastewater to Lake Ft. Gibson:2006 Lognormal Distribution Total-Phosphorus			
Large Wastewater Parameters, Log Transformed			
Ln(X, kg/day)		Arithmetic (X, kg/day)	
N =	364		
μ =	5.931331	E(X)=	378.4626
σ^2 =	0.009572	V(X)=	1377.662
σ =	0.097838	s(X)=	37.11687
CV =	0.016495	CV(X)=	0.098073
Min=	5.752313	Min(X)=	314.9183
Max=	6.127864	Max(X)=	458.4556
		1-sided, α=	0.05
		p (1-α)=	0.95
		Arg (Ø)=	0.95
		r^2=	0.95
		z(p)=	1.645
Maximum Daily Load Parameters			
Upper Limit		X(p)=	442.4268
Variability Factor		VF(p)=X(p)/E(X)=	1.16901
Existing Average		E(x)=	378.4626
% Removal		%R=	45%
Long Term Average		LTA= E(X*)*(1-%R)=	208.1544
Max Daily Load		MDL= LTA*VF(p)=	243.3347

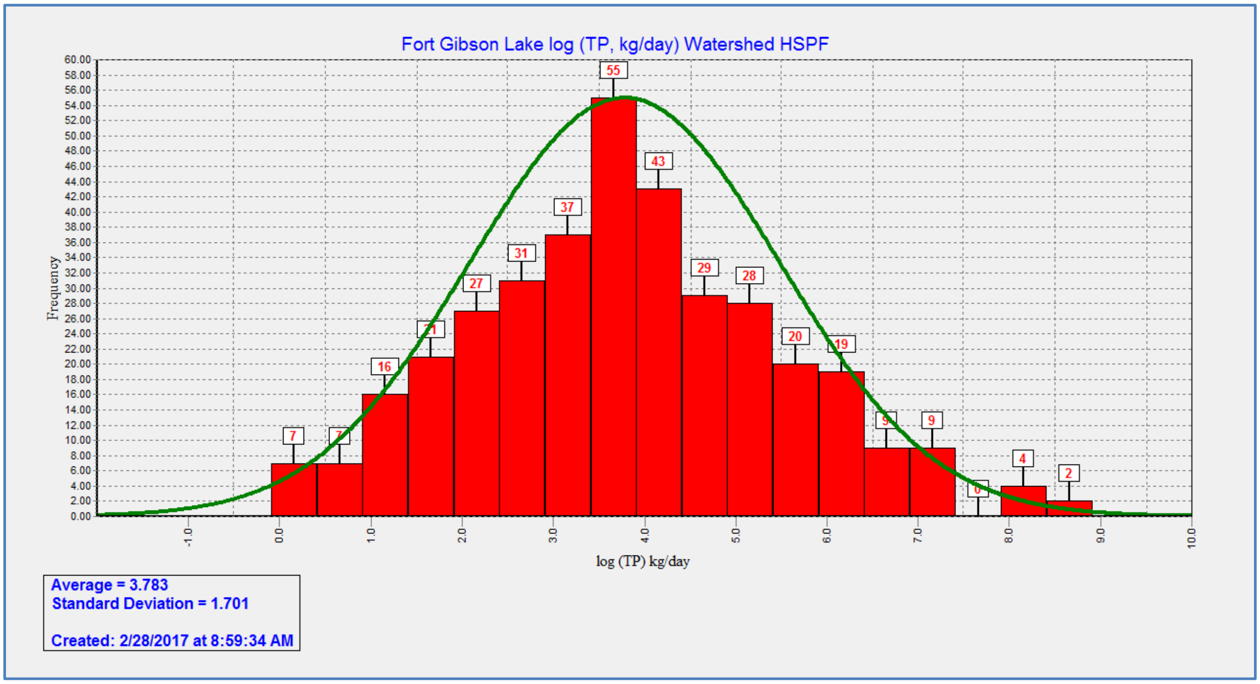

Figure 1 - Histogram of watershed runoff from HSPF model for daily average log transformed TP data from 2006 drought year.

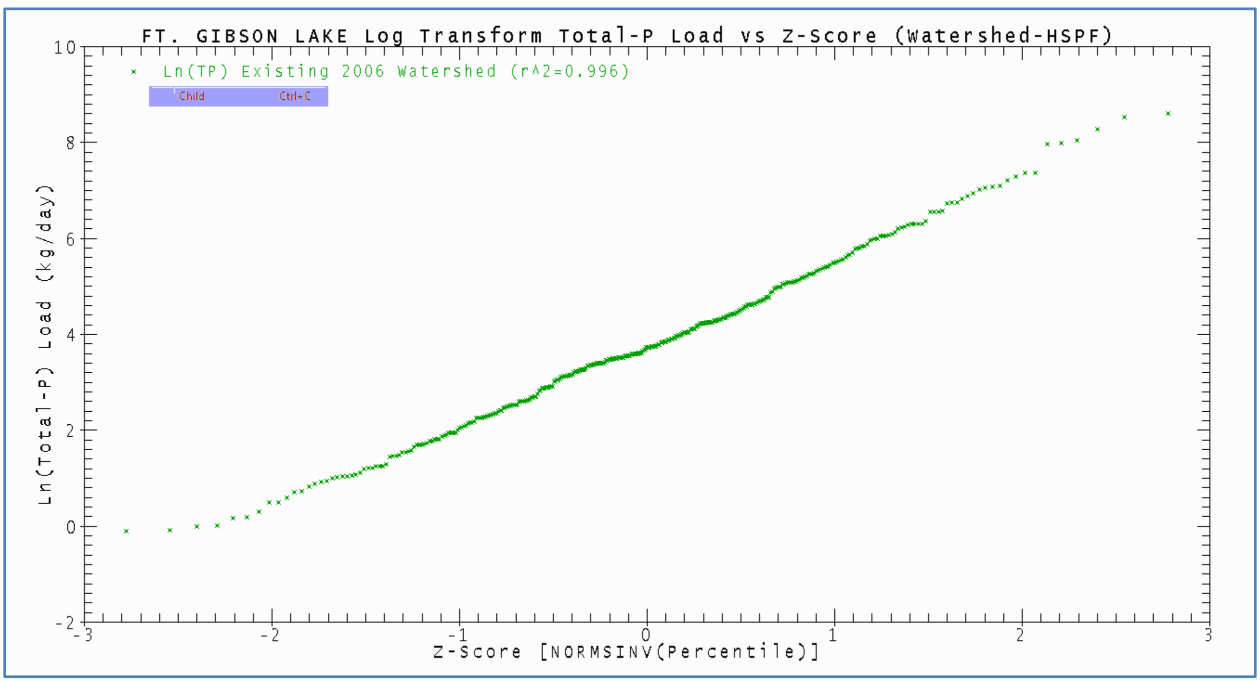

Figure 2 - Probability plot of watershed runoff from HSPF model for daily average log transformed TP data from 2006 drought year (r^2=0.996)

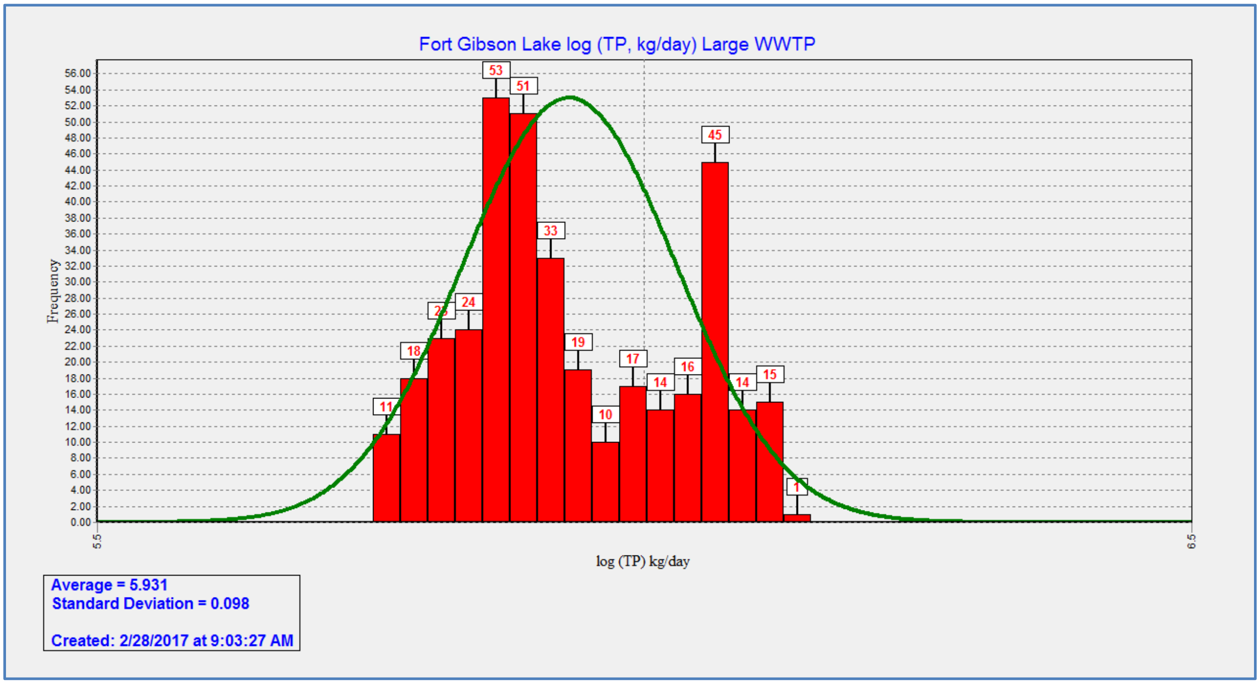

Figure 3 - Histogram of large wastewater effluent load for daily average log transformed TP data from 2006 drought year.

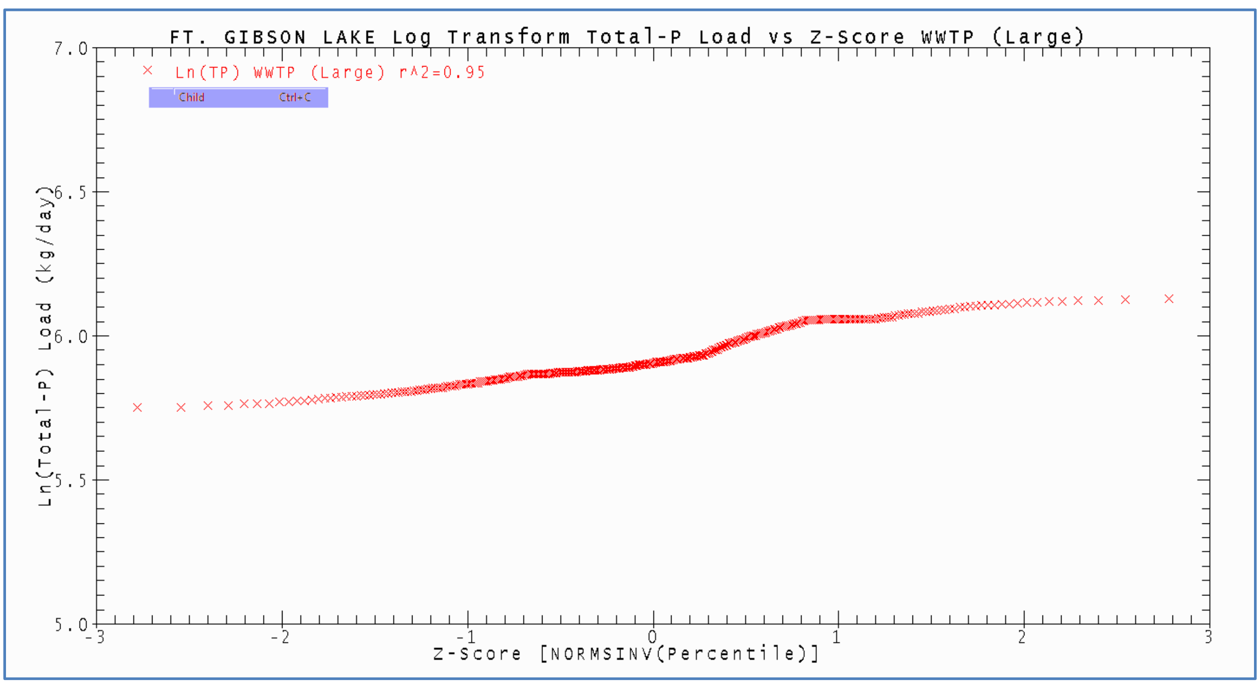

Figure 4 - Probability plot of large wastewater effluent load for daily average log transformed TP data from 2006 drought year (r^2=0.95)

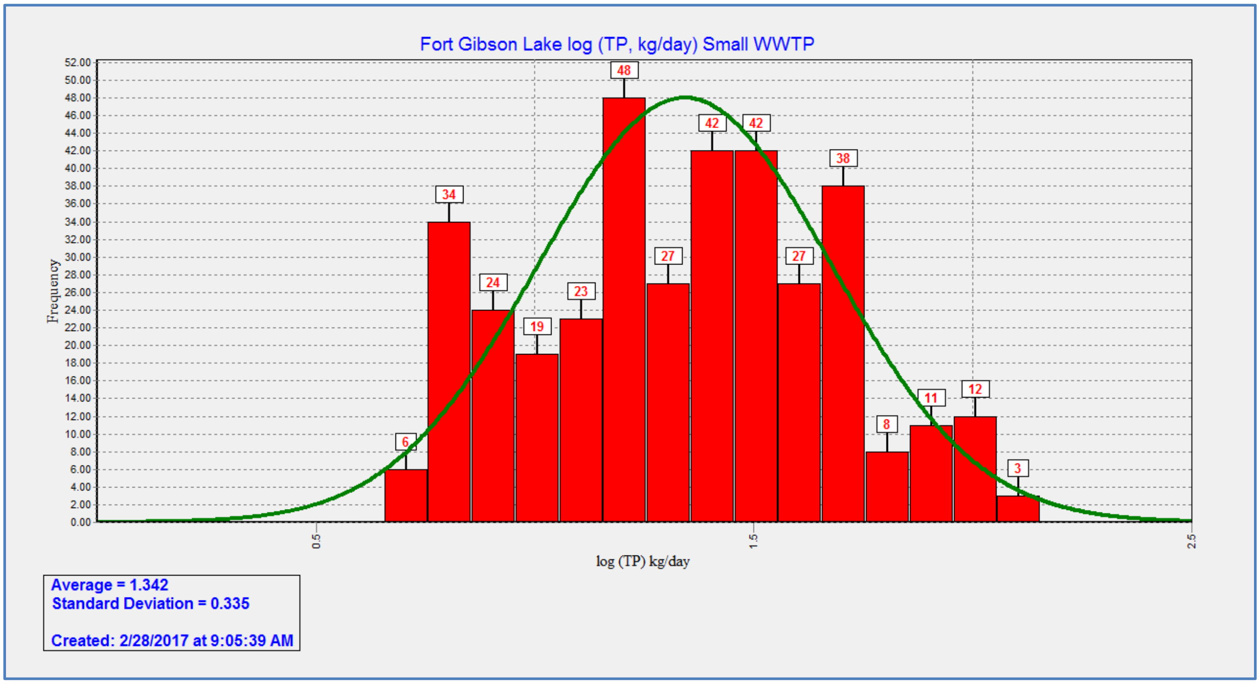

Figure 5 - Histogram of small wastewater effluent load for daily average log transformed TP data from 2006 drought year.

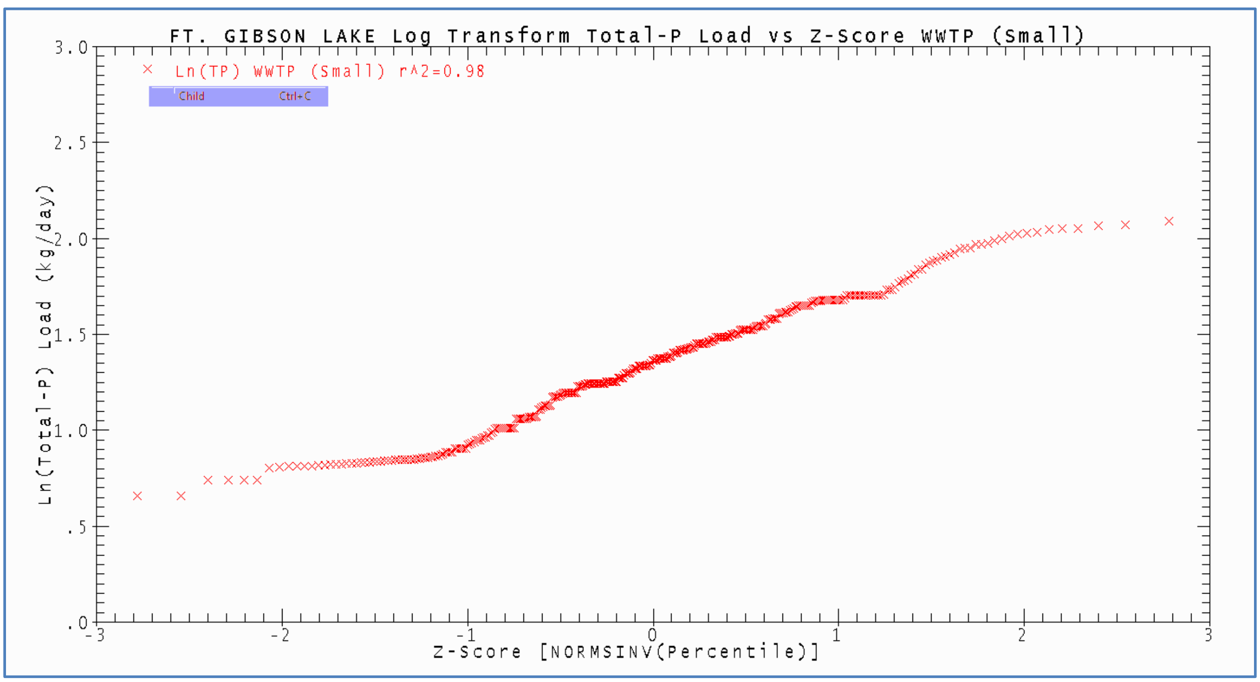

Figure 6 - Probability plot of small wastewater effluent load for daily average log transformed TP data from 2006 drought year (r^2=0.98)

I-3 DELTA LOGNORMAL DISTRIBUTION FOR MAXIMUM DAILY LOADS

I-3.1 Inflow from Lake Hudson as Upstream Boundary to Neosho River and Upper Fort Gibson Lake

The time series of flow records from Lake Hudson into the Neosho River for 2005-2007 (Figure 7) clearly shows the effect of the extreme drought year conditions experienced during 2006 that affected the Lower Neosho River watershed, other areas of Oklahoma and several states of the Central Plains (Tortorelli, 2008; Sandbo et al., 2008). Time series data are presented in Figure 8 only for 2006 to show the persistent pattern of a minimum flow release level related to the periodicity of Grand Lake Development Authority (GRDA) hydropower operations for the Robert S. Kerr Dam, as shown in Figure 9. As can be seen in Figure 10, the histogram of flow data shows significant skewness at the low end of the distribution.

Log transformed flow and loading data of the upstream boundary inflow from Lake Hudson exhibits considerable skewness at the low end of the flow and load distributions. The assumption that the lognormal distribution can represent this external source of pollutant loading to Fort Gibson Lake is, therefore, not appropriate for the 2006 data set used to specify the upstream inflow boundary for the model. The pronounced skewness of flow and pollutant loads at the low end of the data sets from the Lake Hudson inflow suggests, however, that with censoring of the load data based on a minimum load that accounts for the skewness, the delta lognormal distribution may be an appropriate representation of pollutant loads for the Lake Hudson inflow during the drought year conditions of 2006.

As can be seen in the time series (Figure 8) and histogram (Figure 10) for the 2006 inflow data from Lake Hudson, the pattern of data from Lake Hudson shows a remarkable similarity to the censored and non-censored chemical pollutant data from Kahn and Rubin (1989) used to illustrate the delta lognormal distribution (see Figure 11 and Figure 12). Although flow and pollutant loading data are not defined by a “detection limit”, the 2006 flow data in Figure 8 clearly shows a pattern of maintaining a minimum flow release from Lake Hudson to the Neosho River. The visual similarity between the chemical data presented in Kahn and Rubin (1989) and the inflow from Lake Hudson suggests that the inflow load data may be represented as a mix of censored (i.e., less than minimum load) and non-censored (greater than minimum load) data using the delta lognormal distribution as the statistical basis for MDL calculations.

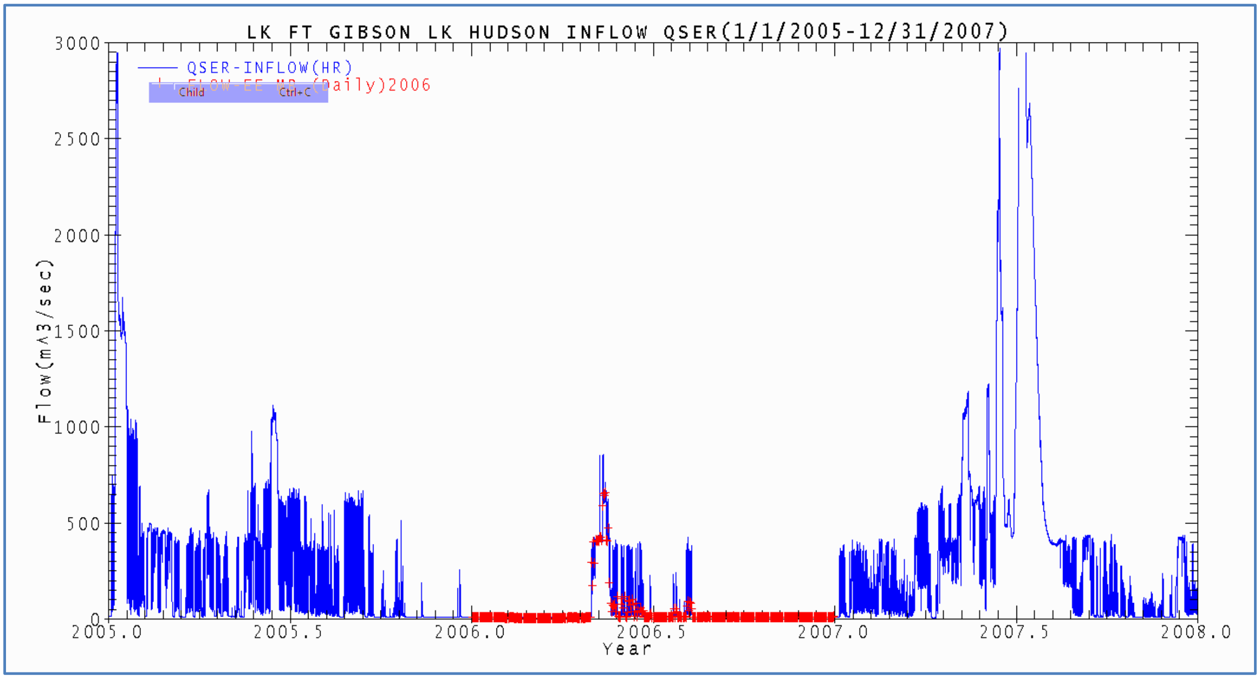

Figure 7 - Time series of upstream boundary inflow from Lake Hudson, 2005-2007. Red + markers identify daily average data for 2006 drought year. Blue line is hourly data.

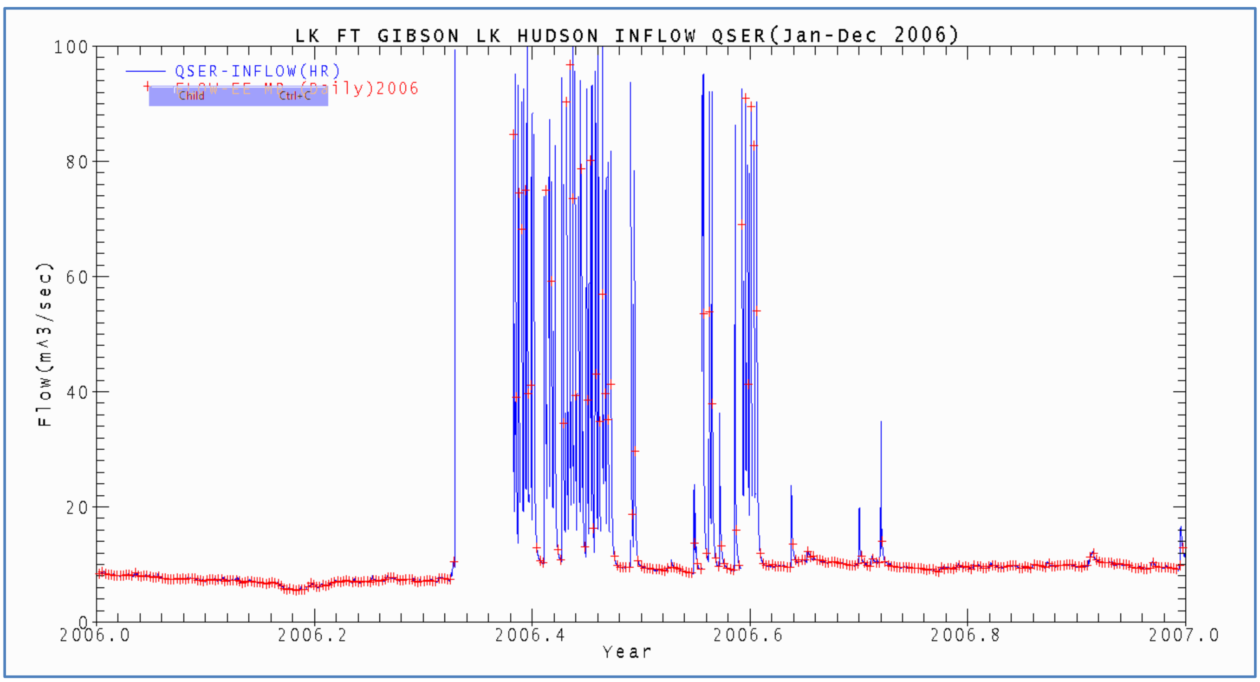

Figure 8 - Time series of upstream boundary inflow from Lake Hudson, Jan-Dec 2006, Red + markers identify daily average data from 2006 drought year. Blue line is hourly data.

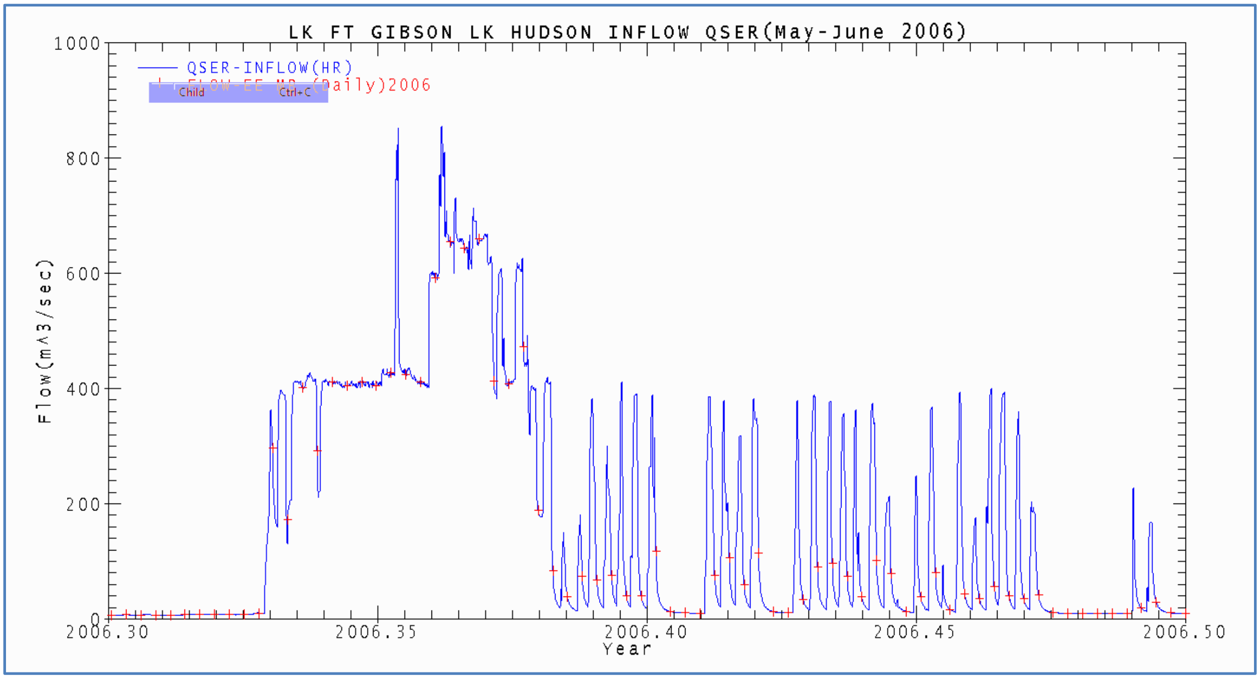

Figure 9 - Detail of time series of upstream boundary inflow from Lake Hudson, 2006, Red + markers identify daily average data from 5/1 -6/30 during 2006 drought year. Blue line is hourly data input to EFDC model. Hourly data shows periodicity of releases for hydropower operations.

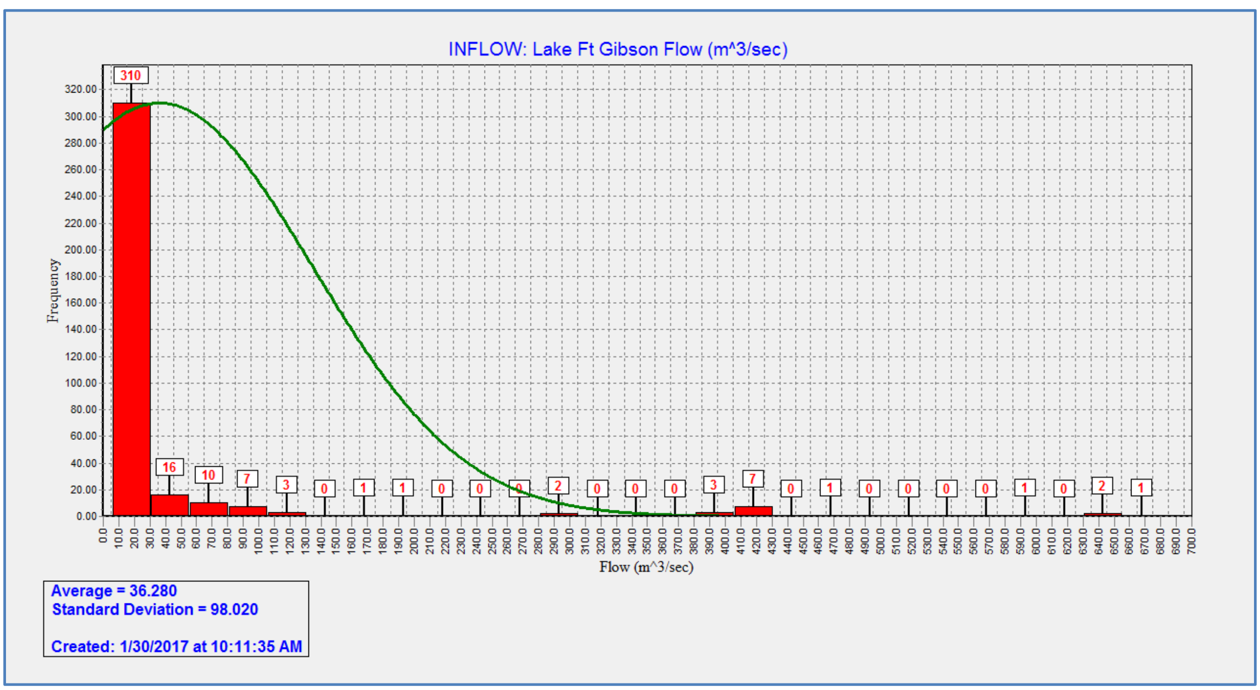

Figure 10 - Histogram of upstream boundary inflow from Lake Hudson for daily average data from 2006 drought year. Flow data are untransformed and shows large skew at low end of distribution with 85% (310 of 365) of the observations less than 25 cms.

INDUSTRIAL WATER POLLUTION CONTROL REGULATIONS [37] 137

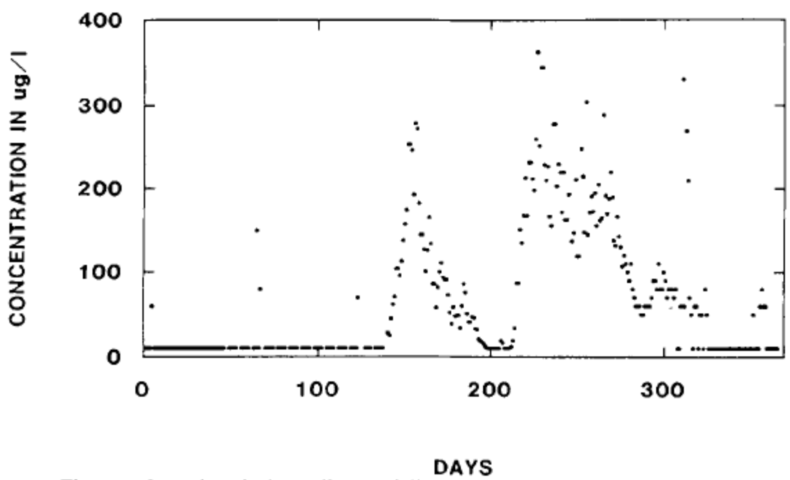

Fig. 4. Organic priority pollutant daily effluent concentrations plant B.
Note: nondetections are at the detection limit of 10 ug l^{-1}.

Figure 11 - Organic priority pollutant daily effluent concentration for plant B. Time series shows observations above and below detection limit (10 µg/L). Source: Kahn and Rubin (1989).

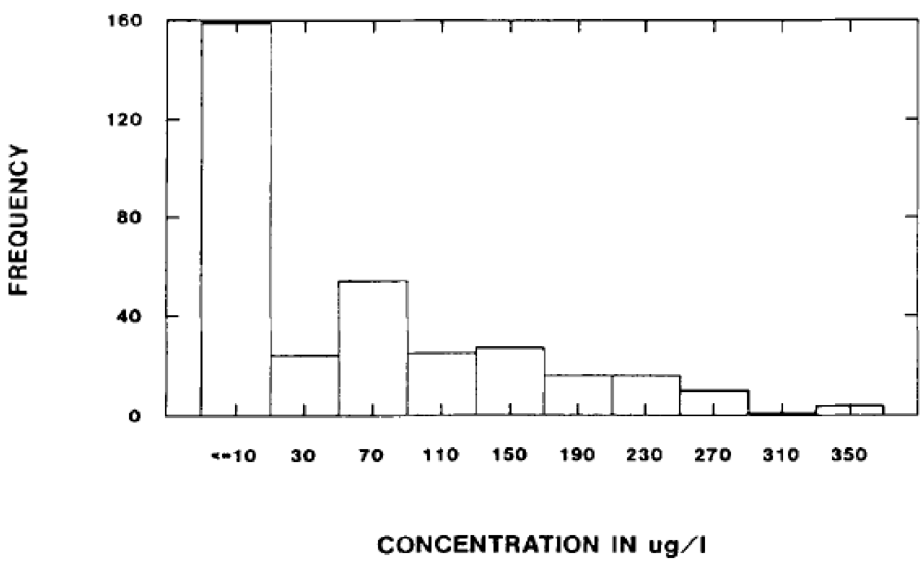

Fig. 7. Organic priority pollutant frequency distribution plant B.

Figure 12 - Histogram of organic priority pollutant daily effluent concentration for plant B. Left peak in histogram shows frequency of observations below detection limit (10 µg/L). Source: Kahn and Rubin (1989).

I-3.2 Equations and Parameters of the Delta Lognormal Distribution

The delta lognormal distribution is a straight forward extension of the lognormal distribution, where the data set is represented as a mix of non-censored data and zeros (or censored data). Observations greater than the detection limit for censored data are described by the lognormal distribution and the distribution of records lower than the detection limit is represented with a discrete probability of recording a measurement at, or below, the detection limit. Owen and De Rouen (1980) recommended use of the delta lognormal distribution for pollutant data sets that included zero values and censored data (i.e., measurements reported as less than the detection limit). The methodology was first adopted by EPA for the development of effluent guidelines for the Organic Chemicals, Plastics and Synthetic Fibers industry (EPA, 1987) and subsequently applied for effluent guidelines for other industry groups (e.g., Iron and Steel: EPA, 2002). The methodology, equations, and parameters used for the delta lognormal distribution to derive maximum daily loads, briefly described in Appendix E of EPA (1991), are described in detail by Kahn and Rubin (1989). The equations for the delta lognormal distribution are presented herein with the parameters used to develop the maximum daily load calculations.

For a data set of size (N) characterized by a mix of censored and non-censored load data, two new parameters are required for MDL calculations based on the delta lognormal distribution. The first new parameter (D) is defined as the censored value (or detection limit) of the data set. The second new parameter (δ) is defined as the proportion of the number of data records (r), where the observed value is less than or equal to the censored value ($\delta = r/N$).

Following the notation of Kahn and Rubin and EPA (1991), X^* is used to define the random variable for the modified delta distribution. The mean $E(X^*)$, variance $V(X^*)$, and upper percentile limit (X^*_p) of the modified random variable 'X^*' expressed in the measured units of the original untransformed data set are given as follows:

$$E(X^*) = \delta D + (1-\delta)\exp\,(\mu + 0.5\sigma^2)$$ ------ Equation 11

$$V(X^*) = (1-\delta)\exp(2\mu + \sigma^2)\,[\exp(\sigma^2) - (1-\delta)] + \delta(1-\delta)D[D - 2\exp(\mu + 0.5\sigma^2)]$$ Equation 12

$$X_p{}^* = \max\,[D, \exp(\mu + Z^*\sigma)]$$ ------ Equation 13

$$Z^* = \emptyset^{-1}[(p-\delta)/(1-\delta)]$$ ------ Equation 14

The probability value of the argument for the function $\emptyset^{-1}(\,)$ is based on the censored proportion of the input data set and the desired probability for the confidence interval of the distribution. The function $\emptyset^{-1}(\,)$ is the inverse of the standard normal cumulative distribution function. The probability value of the argument can be input to the Microsoft Excel function NORMSINV to obtain the parameter value for the Z-score (Z^*) as a function of the desired percentile (p) for the confidence interval of the distribution.

Note that for the case where $\delta = 0$ the value for $Z^* = 2.326$ for p=0.99 and the value for $Z^* = 1.645$ for p=0.95. As an example of the calculation of the argument and the Z-score parameter based on the

95th percentile (p=0.95) and a data set where 80% of the records are marked as censored data ($\delta = 0.8$), the argument for the function $\emptyset^{-1}(\ )$ is $[(0.95 - 0.8)/(1 - 0.8)] = 0.75$ and the value returned from NORMSINV (0.75) is$Z^{*} = 0.6745$.

The daily maximum variability factor (VF_p) for the pth percentile of the standard normal distribution, based on the ratio of high values to average values of the non-censored lognormal distribution, is defined by Kahn and Rubin (1989) and EPA (1991) as:

$VF_p^{*} = X_p^{*}/E(X^{*})$ -- Equation 15

Following the methodology for the lognormal distribution, the variability factor is used to calculate the MDL from the expected value of the existing average load [E(X*)] and the percent reduction (%R) of the existing load needed to attain compliance with water quality targets. E(X*) accounts for the minimum value for the censored limit, the proportion of the total load data that is defined as censored and non-censored data, and the parameter values of the distribution of the log transformed load censored load data. The MDL is computed from the expected value of the existing load, the required removal percentage for compliance with water quality targets, and the temporal variability factor as follows:

$MDL = E(X^{*})(1 - \%R)VF_p$ -- Equation 16

I-3.3 Delta Lognormal Representation of Lake Hudson Inflow Loads

The pronounced skewness of the pollutant loads at the low end of the loading data from the Lake Hudson inflow suggests that, with censoring of the load data for a minimum value that accounts for the skewness, the delta lognormal distribution may be an appropriate representation of pollutant loads for the inflow from Lake Hudson during the drought conditions of 2006. Load data derived from flow and water quality observations of TP for the inflow from Lake Hudson are used as an example data set for TP to show that the delta lognormal distribution for the inflow from Lake Hudson is an appropriate assumption for MDL calculations for Fort Gibson Lake.

Data for Total Phosphorus (TP) are analyzed and presented to illustrate the use of the delta lognormal distribution to represent the TP time series for 2006 for calculations of distribution parameters and the maximum daily load for TP from Lake Hudson. Parameter values of the delta lognormal distribution were estimated and histogram and probability plots of the non-censored log transformed TP load data sets were prepared to evaluate the validity of the delta lognormal distribution as a representation of the inflow loads from Lake Hudson to the Neosho River and Upper Fort Gibson Lake.

The 2005-2006 time series of TP load data for the outflow from Lake Hudson (Figure 13) shows the effect of low-flow drought conditions on the outflow of TP from Lake Hudson to the Neosho River in 2006 (displayed on the right-hand side of the graph). The persistent pattern of the low TP load exported from Lake Hudson in 2006 is shown in Figure 14. Using a few iterations for evaluation of the lognormal approximation for non-censored TP load data, the censored value for the minimum TP load was determined to be D=200 kg/day. The histogram of the TP loading data (Figure 15) for 61 non-censored observations approximates a bell-shaped curve and the probability distribution of the log transformed non-censored TP load data (Figure 16) is consistent with a linear relationship expected for the lognormal portion of the delta lognormal distribution.

After censoring of TP load data for the inflow from Lake Hudson with a minimum load (D=200 kg/day), the probability plot of the log transformed non-censored TP load data shows a very good approximation to a linear relationship (r^2=0.977). This ensures the validity of the delta lognormal distribution for MDL calculations for the inflow from Lake Hudson. As flow is common to all loads derived from observed flow and water quality data for the upstream boundary inflow from Lake Hudson, Total Phosphorus (TP), Total Nitrogen (TN), Total Organic Carbon (TOC), and Total Suspended Solids (TSS) all display similar mixes of censored and non-censored load data that can be represented by the delta lognormal distribution for MDL calculations.

The parameters and MDL calculations derived from the delta lognormal equations are presented in Table 4 for the inflow from Lake Hudson as an example for TP loading. With 83% of the total data set (N=365) defined by r=304 censored data records less than the censored load of 200 kg/day, the Z-score for the 95th percentile probability is calculated with the Excel function NORMSINV as Z^*= 0.5267. With the TP load for the upper 95th percentile confidence limit [X*(p)] of 2,622.5 kg/day and an expected value [E(X*)] of 565.7 kg/day for the existing TP load, the daily variability factor [VF^*(p)] is 4.635. The EFDC lake model was used to determine that an overall 45% reduction of external loading to the lake was required for compliance with water quality targets for anoxic volume, trophic state index, and turbidity. For more detail on the water quality targets used for the TMDL determinations, refer to Section 2 of the TMDL report for a discussion of Oklahoma water quality standards and the water quality targets for Fort Gibson Lake.

For the MDL calculation, the long-term average reduced load for the inflow from Lake Hudson (LTA) is estimated from the expected value [E(X*) =565.7 kg/day) and 45% removal for compliance with water quality targets as LTA=565.7 x (1-0.45) = 311.2 kg/day. Based on the LTA of 311.2 kg/day and the variability factor [VF^*(p) =4.635], the MDL for the inflow from Lake Hudson is estimated as MDL = 311.2 x 4.635 = 1,442.4 kg/day.

Similar calculations derived from the lognormal distribution equations and parameters are presented for TP for watershed runoff and small and large wastewater loading in Table 1, Table 2 and Table 3. The LTA for TP for Fort Gibson Lake is derived as the sum of the reduced LTAs calculated for each source and the TMDL for TP for Fort Gibson Lake is derived as the sum of the MDLs calculated for each source as follows:

$$TMDL = MDL\ (Inflow) + MDL\ (Watershed) + \ MDL\ (Small\ WWTP) + \ MDL\ (Large\ WWTP)$$
-- Equation 17

Table 4 - Delta Lognormal Parameters and Estimation of Maximum Daily Load for Total Phosphorus Load from Lake Hudson Inflow to Neosho River in 2006

Lake Hudson Inflow to Neosho River: 2006 Delta Lognormal Distribution Total-Phosphorus			
Censored Parameters			
D =	200	Min Load (kg/day)	
r=	304	Obs <= D	
N=	365	Total Obs	
ð=r/N	0.832877	Fraction censored data	
Non-Censored Parameters, Log Transformed			
Ln (X, kg/day)		Arithmetic (X, kg/day)	
k=N-r	61	N_Obs > D	
μ=	7.435875	E(X*)=	565.7658
σ^2=	0.685175	V(X*)=	1605094
σ=	0.827753	s(X*)=	1266.923
CV=	0.111319	CV(X*)=	2.239306
Min	5.794635	Min(X*)=	328.5323
Max	8.85897	Max(X*)=	7037.233
		1-sided, a=	0.05
		Probability,p=	0.95
		Arg (Ø)=(p-ð)/(1-ð)	0.70082
		r^2=	0.977
		Z*p =	0.526759
Maximum Daily Load Parameters			
		X*(p)=	2622.547
		VF(p)=X*(p)/E(X*)=	4.635393
		Existing Avg: E(X*)=	565.7658
		% Removal: %R=	45%
		LTA= E(X*)*(1-%R)=	311.1712
		MDL= LTA*VF(p)=	1442.401

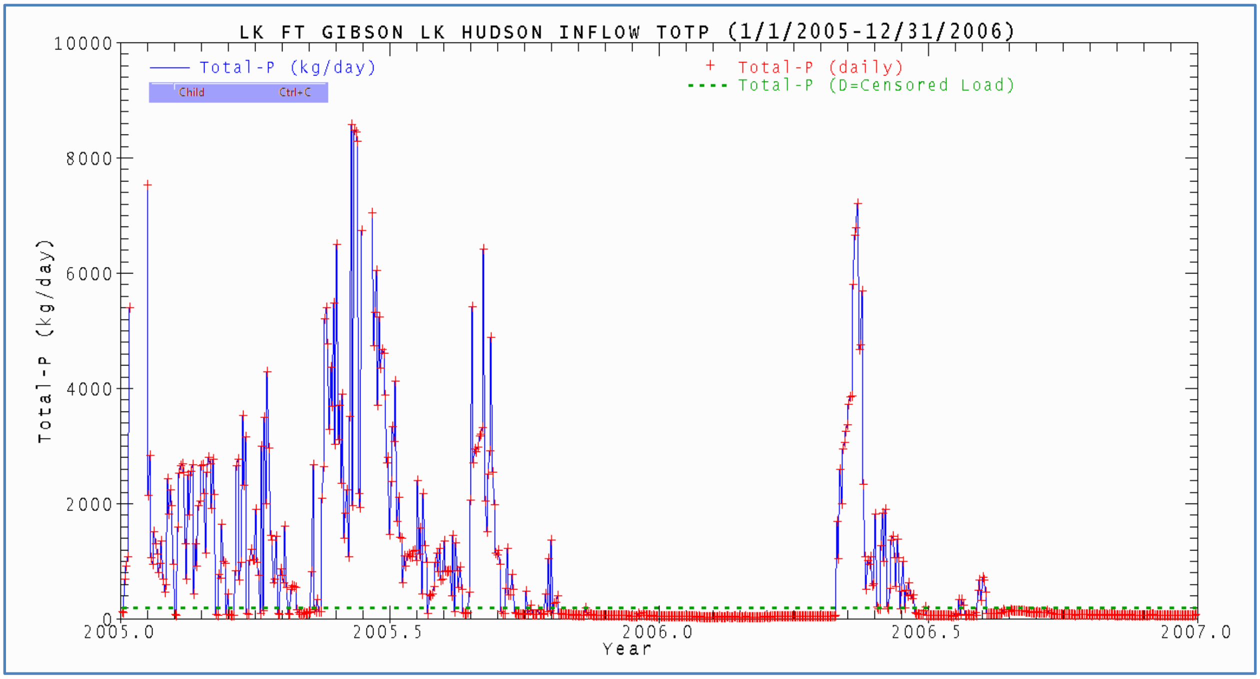

Figure 13 - Time series of TP load from Lake Hudson Inflow for 2005-2006

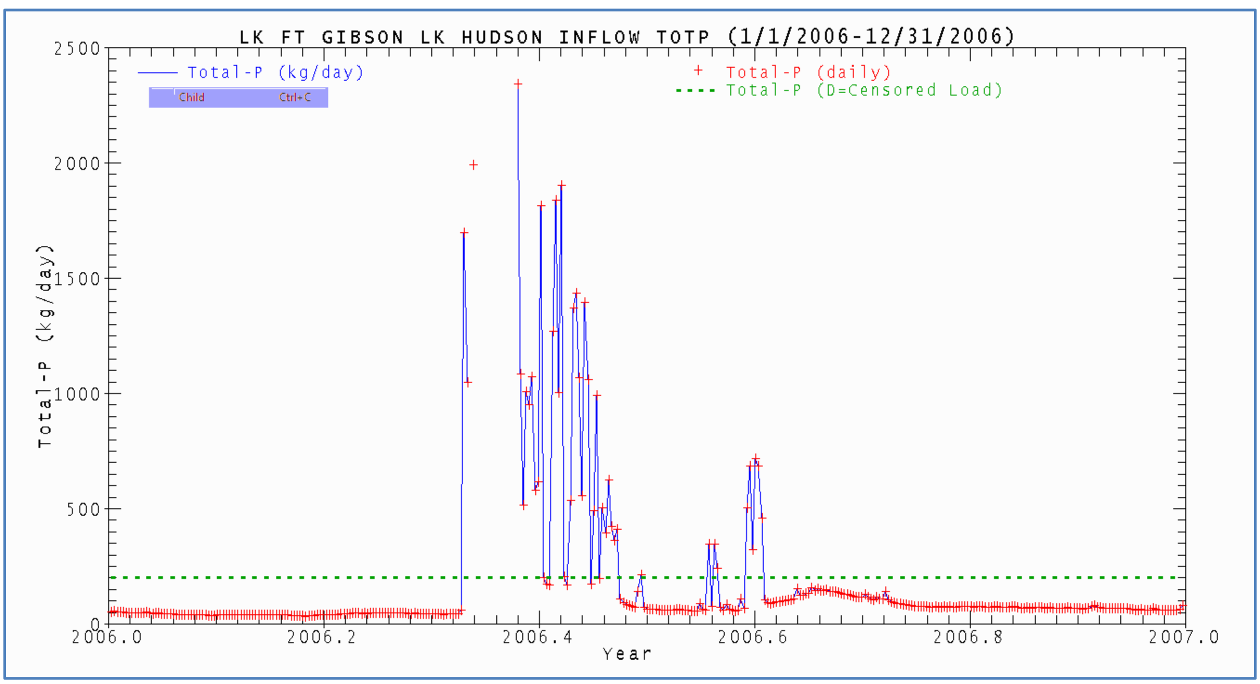

Figure 14 - Time series of TP load from Lake Hudson inflow for 2006. The dashed green line shows the censored minimum load (D=200 kg/day) determined for TP.

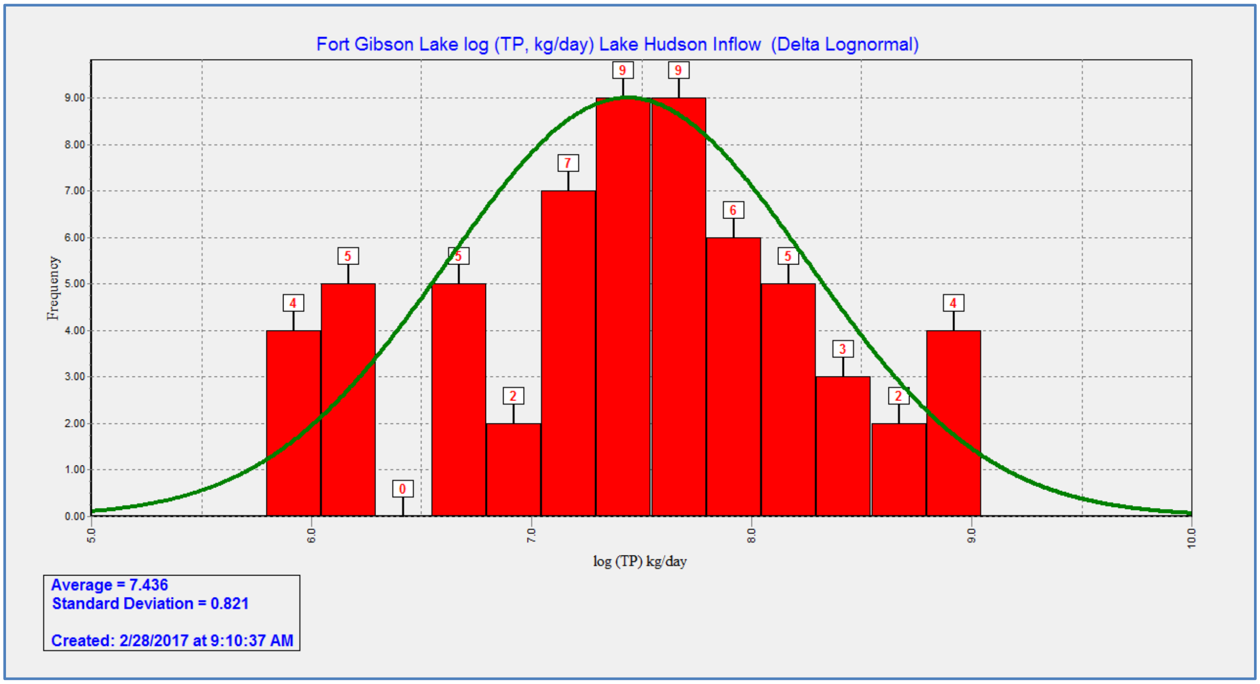

Figure 15 - Histogram of upstream boundary inflow from Lake Hudson for non-censored daily average log transformed TP data from 2006 drought year.

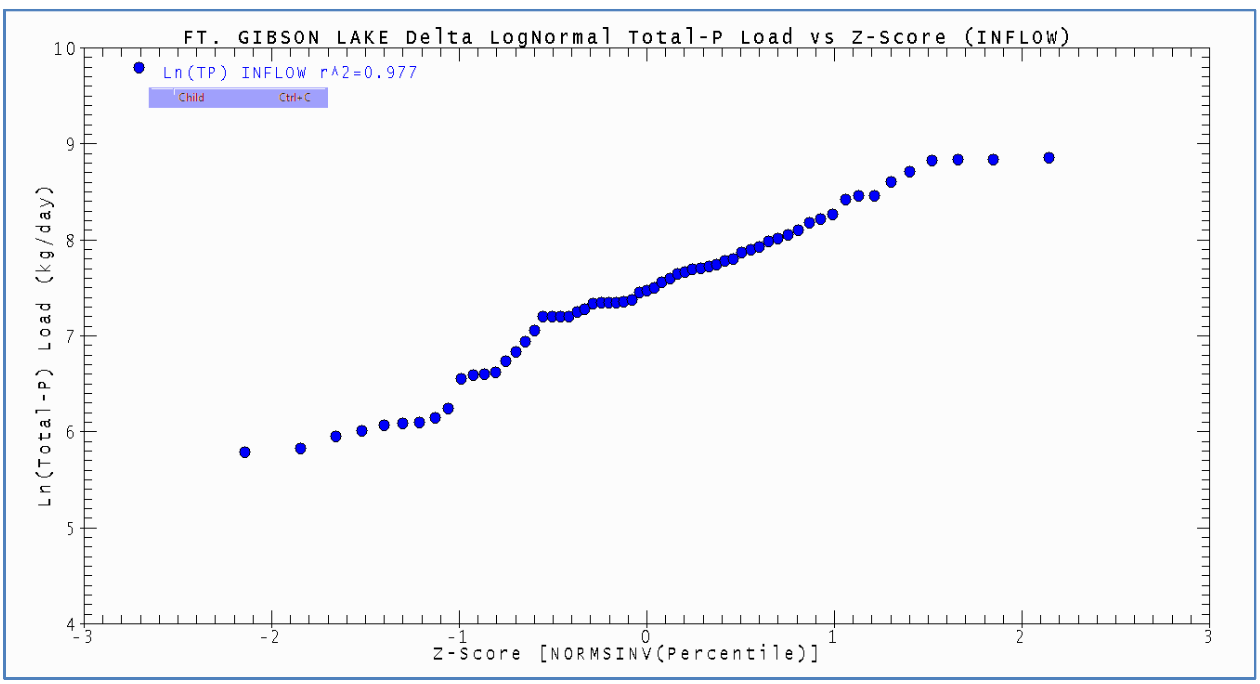

Figure 16 - Probability plot of upstream boundary inflow from Lake Hudson for non-censored daily average log transformed TP data from 2006 drought year (r^2=0.977)

I-4 EXISTING LONG-TERM AVERAGE LOADS AND MAXIMUM DAILY LOADS FOR TP, TN, TOC, AND TSS

Total Phosphorus. Table 1 through Table 4 presents the statistics derived for the loading analysis of Total Phosphorus and Table 5 summarizes the parameters and maximum daily load calculations for the source categories of Total Phosphorus from (a) inflow from Lake Hudson, (b) watershed runoff, and (c) small and (d) large wastewater dischargers. The maximum daily loads derived for each external source term are summed to determine the total maximum daily load for Total Phosphorus loading to Fort Gibson Lake. The inflow from Lake Hudson is described by the delta lognormal distribution and the source terms for the watershed and small and large wastewater facilities are described by lognormal distributions. Trend-line regression of the probability plots demonstrate strong linear relationships with r^2 values of 0.977 for the inflow from Lake Hudson, 0.996 for watershed runoff, 0.98 for small wastewater facilities, and 0.95 for large wastewater facilities.

Table 6 presents a summary of the maximum daily load allocations for TP for the inflow from Lake Hudson, watershed runoff, and wastewater facilities. The Total Maximum Daily Load for TP of 2,087.6 kg/day is allocated to each source term as a Load Allocation (LA) or Waste Load Allocation (WLA) where the allocation is proportional to the percentage contribution of the source to the total existing load. For the six NPDES wastewater dischargers, there are three small and three large facilities where small (< 1 MGD) and large (> 1 MGD) are defined by the effluent flow rate.

The methodology, equations, parameters, and procedures described for the TP example calculations based on the lognormal and delta lognormal distributions were applied for determining the maximum daily loads for TN, TOC, and TSS. Statistics derived for the loading analysis for the source categories from (a) inflow from Lake Hudson, (b) watershed runoff, and (c) small and (d) large wastewater dischargers and summary data tables of the loading parameters and MDL calculations for each water quality variable are presented below for TN, TOC and TSS.

Total Nitrogen. Maximum daily loads for the inflow from Lake Hudson are derived with the equations and parameters of the delta lognormal distribution while maximum daily loads for watershed runoff and wastewater sources are based on the lognormal distribution. Table 7 through Table 10 present the statistics derived for the loading analysis of TN for watershed runoff, small and large wastewater facilities, and the inflow from Lake Hudson to the Neosho River. Table 11 presents a summary of the parameters and maximum daily loads derived for TN for each source and Table 12 presents a summary of the load and wasteload allocations for TN for each source.

Total Organic Carbon. Maximum daily loads for the inflow from Lake Hudson are derived with the equations and parameters of the delta lognormal distribution while maximum daily loads for watershed runoff and wastewater sources are based on the lognormal distribution. Table 13 through Table 16 presents the statistics derived for the loading analysis of TOC for watershed runoff, small and large wastewater facilities, and the inflow from Lake Hudson to the Neosho River. Table 17 presents a summary of the parameters and maximum daily loads derived for TOC for each source and Table 18 presents a summary of the load and wasteload allocations for TOC for each source.

Total Suspended Solids. Maximum daily loads for the inflow from Lake Hudson are derived with the equations and parameters of the delta lognormal distribution and maximum daily loads for wastewater sources are based on the lognormal distribution. The time series of TSS loading from the HSPF watershed model for 2006 is shown in Figure 17 and Figure 18 shows the probability plot for the lognormal distribution for watershed runoff.

As can be seen in the probability plot, there is significant skewness at the very low side of the distribution. Analysis of the lognormal distribution showed that the correlation for TSS (r^2 =0.95) was not as good as the r^2 coefficients for lognormal watershed loading of TP (r^2 =0.996), TN (r^2 =0.984) and TOC (r^2 =0.989). In addition to higher correlations for the probability plots, the probability distributions of TP, TN, and TOC were not characterized by pronounced skew at the lower end of the load distributions. The delta lognormal distribution, instead of less than satisfactory relationship based on the lognormal distribution, was used to describe TSS loading from watershed runoff. A few iterations quickly determined that a very small censored load of D=1.0 kg/day greatly improved the relationship of the non-censored portion of the TSS load data. With 26% of the watershed data defined by a censored TSS load of 1.0 kg/day, transformation of the non-censored load data results in a significant improvement of the linear relationship of the lognormal probability plot to an r^2 value of 0.992 (see Figure 19).

Table 19 through Table 22 presents the statistics derived for the loading analysis of TSS for watershed runoff, small and large wastewater facilities, and the inflow from Lake Hudson to the Neosho River. Table 23 presents a summary of the parameters and maximum daily loads derived from each source for TSS and Table 24 presents a summary of the load allocations and wasteload allocations for TSS for each source.

Table 5 - Summary of Parameters and Maximum Daily Loads for External Sources of Total Phosphorus to Fort Gibson Lake in 2006

Fort Gibson Lake: Jan-Dec 2006 Total-Phosphorus					
Parameter	Lake Hudson Inflow	Watershed HSPF	Small WWTP	Large WWTP	Total
Min: D (kg/day)	200	0	0	0	
N-r (> D)	61	364	364	364	
r (< D)	304	0	0	0	
N	365	364	364	364	
E(X) (kg/day)	565.8	187.4	4.0	378.5	1,135.7
V(X)	1,605,093.6	604,212.2	2.0	1,377.7	
s(X) (kg/day)	1,266.9	777.3	1.4	37.1	
CV(X)	2.2	4.1	0.3	0.1	
Min(X) (kg/day)	328.5	0.9	1.9	314.9	
Max(X) (kg/day)	7,037.2	5,499.6	8.1	458.5	
Probability,p	0.95	0.95	0.95	0.95	
ð=r/N	0.833	0.000	0.000	0.000	
Arg (Ø) = (p-ð)/(1-ð)	0.701	0.950	0.950	0.950	
R^2=	0.977	0.996	0.980	0.950	
Z(p)	0.527	1.645	1.645	1.645	
X(p) (kg/day)	2,622.5	724.0	6.6	442.4	
VF(p)	4.64	3.86	1.64	1.17	
% Reduction	0.45	0.45	0.45	0.45	0.45
LTA (kg/day)	311.2	103.1	2.2	208.2	624.6
MDL (kg/day)	1,442.4	398.2	3.7	243.3	2,087.6
Lake Hudson	Delta lognormal distribution				
Watershed HSPF	Lognormal distribution				
Small WWTP	Lognormal distribution				
Large WWTP	Lognormal distribution				

Table 6 - Maximum Daily Load Allocations for Fort Gibson Lake: Total Phosphorus

Fort Gibson Lake Total-Phosphorus			% R= 45% TMDL= 2,087.6 kg/day			
Source	Existing E(X) Mean	Existing % Share	LA kg/day	WLA kg/day	LA+WLA kg/day	Margin of Safety
Lake Hudson Inflow	565.8	49.8%	1,039.95	0.0	1,039.9	Implicit
Watershed HSPF	187.4	16.5%	344.56	0.0	344.6	Implicit
Small WWTP	4.0	0.4%	0.00	7.4	7.4	Implicit
Large WWTP	378.5	33.3%	0.00	695.7	695.7	Implicit
Total	**1,135.7**	**100.0%**	**1,384.50**	**703.1**	**2,087.6**	Implicit
NPDES Wastewater						
OK0043907 (S)	3.5	0.31%	0	6.5	6.5	Implicit
OKG380001 (S)	0.3	0.03%	0	0.5	0.5	Implicit
OK0033791 (S)	0.2	0.02%	0	0.4	0.4	Implicit
OK34568-006 (L)	364.7	32.11%	0	670.3	670.3	Implicit
OK0000272 (L)	1.8	0.16%	0	3.3	3.3	Implicit
OK0035149 (L)	12.0	1.06%	0	22.1	22.1	Implicit
Lake Hudson Inflow	Delta lognormal distribution					
Watershed HSPF	Lognormal distribution					
Small WWTP	Lognormal distribution					
Large WWTP	Lognormal distribution					

Table 7 - Lognormal Parameters and Estimation of Maximum Daily Load for Total Nitrogen for Watershed (HSPF) Runoff in 2006

Watershed Runoff to Lake Ft. Gibson: 2006 Lognormal Distribution Total-Nitrogen			
Watershed Parameters, Log Transformed			
Ln(X, kg/day)		Arithmetic (X, kg/day)	
N (Obs)=	364		
μ=	5.856835	E(X)=	1663.876
Var=	3.120141	V(X)=	59936479
StdDev=	1.766392	s(X)=	7741.865
CoeffVar=	0.301595	CV(X)=	4.65291
Min	1.7042	Min(X)=	5.496986
Max	10.00143	Max(X)=	22057.96
		1-sided, a=	0.05
		Probability,p=	0.95
		Arg (Ø)=	0.95
		Z(p)=	1.645
		R^2=	0.9837
Maximum Daily Load Parameters			
		X(p)=	6390.384
		VF(p)=X(p)/E(X)=	3.840661
		Existing Avg: E(X)=	1663.876
		% Removal: %R=	45%
		LTA= E(X*)*(1-%R)=	915.1319
		MDL= LTA*VF(p)=	3514.711

Table 8- Lognormal Parameters and Estimation of Maximum Daily Load for Total Nitrogen for Small Wastewater Loads in 2006

Small Wastewater to Lake Ft. Gibson: 2006 Lognormal Distribution Total-Nitrogen			
Small Wastewater Parameters, Log Transformed			
Ln (X, kg/day)		Arithmetic (X, kg/day)	
N (Obs)=	364		
μ=	3.316529	E(X)=	28.95627
Var=	0.098517	V(X)=	86.80857
StdDev=	0.313873	s(X)=	9.317112
CoeffVar=	0.094639	CV(X)=	0.321765
Min	2.82853	Min(X)=	16.92057
Max	3.782029	Max(X)=	43.90502
		1-sided, a=	0.05
		Probability,p=	0.95
		Arg (Ø)=	0.95
		Z(p)=	1.645
		R^2=	0.909
Maximum Daily Load Parameters			
		X(p)=	46.19402
		VF(p)=X(p)/E(X)=	1.595303
		Existing Avg: E(X)=	28.95627
		% Removal: %R=	45%
		LTA= E(X*)*(1-%R)=	15.92595
		MDL= LTA*VF(p)=	25.40671

Table 9- Lognormal Parameters and Estimation of Maximum Daily Load for Total Nitrogen for Large Wastewater Loads in 2006

Large Wastewater to Lake Ft. Gibson:2006 Lognormal Distribution Total-Nitrogen			
Large Wastewater Parameters, Log Transformed			
Ln (X, kg/day)		Arithmetic (X, kg/day)	
N (Obs)=	364		
μ=	6.495119	E(X)=	667.4454
Var=	0.016677	V(X)=	7491.692
StdDev=	0.12914	s(X)=	86.55456
CoeffVar=	0.019883	CV(X)=	0.12968
Min	6.213631	Min(X)=	499.5117
Max	6.640205	Max(X)=	765.2522
		1-sided, a=	0.05
		Probability,p=	0.95
		Arg (Ø)=	0.95
		Z(p)=	1.645
		R^2=	0.814
Maximum Daily Load Parameters			
		X(p)=	818.5663
		VF(p)=X(p)/E(X)=	1.226417
		Existing Avg: E(X)=	667.4454
		% Removal: %R=	45%
		LTA= E(X*)*(1-%R)=	367.095
		MDL= LTA*VF(p)=	450.2115

Table 10- Delta Lognormal Parameters and Estimation of Maximum Daily Load for Total Nitrogen from Lake Hudson inflow to Neosho River in 2006

Hudson Lake Inflow to Neosho River: 2006 Delta Lognormal Distribution Total-Nitrogen			
Censored Parameters			
D =	1000	Min Load (kg/day)	
r=	294	Obs <= D	
N=	365	Total Obs	
ð=r/N	0.805479	Fraction censored data	
Non-Censored Parameters, Log Transformed			
Ln (X, kg/day)		Arithmetic (X, kg/day)	
k=N-r	71	Obs > D	
μ=	9.27084	E(X*)=	5012.187
Var=	1.421625	V(X*)=	3.53E+08
StdDev=	1.192319	s(X*)=	18779.45
CoeffVar=	0.12861	CV(X*)=	3.746758
Min	6.999443	Min(X*)=	1096.023
Max	11.14187	Max(X*)=	69000.55
		1-sided, a=	0.05
		Probability,p=	0.95
		Arg (Ø)=(p-ð)/(1-ð)	0.742958
		Z*p =	0.652491
		R^2=	0.9564
Maximum Daily Load Parameters			
		X*(p)=	23128.43
		VF(p)=X*(p)/E(X*)=	4.614439
		Existing Avg: E(X*)=	5012.187
		% Removal: %R=	45%
		LTA= E(X*)*(1-%R)=	2756.703
		MDL= LTA*VF(p)=	12720.64

Table 11- Summary of Parameters and Maximum Daily Loads for External Sources of Total Nitrogen (TN) to Fort Gibson Lake in 2006

Fort Gibson Lake: Jan-Dec 2006 Total-Nitrogen					
Parameter	Hudson Lk Inflow	Watershed (Trib+NPS)	Small WWTP	Large WWTP	Total
Min: D (kg/day)	1,000	0	0	0	
N-r (> D)	71	364	364	364	
r (< D)	294	0	0	0	
N	365	364	364	364	
E(X) (kg/day)	5,012.2	1,663.9	29.0	667.4	7,372.5
V(X)	3.5267E+08	5.9936E+07	86.8	7,491.7	
s(X) (kg/day)	18,779.5	7,741.9	9.3	86.6	
CV(X)	3.75	4.65	0.32	0.13	
Min(X) (kg/day)	1,096.0	5.5	16.9	499.5	
Max(X) (kg/day)	69,000.6	22,058.0	43.9	765.3	
Probability,p	0.95	0.95	0.95	0.95	
ð=r/N	0.805	0.000	0.000	0.000	
Arg (Ø)=(p-ð)/(1-ð)	0.743	0.950	0.950	0.950	
R^2=	0.956	0.984	0.909	0.814	
Z(p)	0.652	1.645	1.645	1.645	
X(p) (kg/day)	23,128.4	6,390.4	46.2	818.6	
VF(p)	4.61	3.84	1.60	1.23	
% Reduction	45%	45%	45%	45%	45%
LTA (kg/day)	2,756.7	915.1	15.9	367.1	4,054.9
MDL (kg/day)	12,720.6	3,514.7	25.4	450.2	16,711.0
Hudson Lake Inflow	Delta lognormal distribution				
Watershed	Lognormal distribution				
Small WWTP	Lognormal distribution				
Large WWTP	Lognormal distribution				

Table 12- Maximum Daily Load Allocations for Fort Gibson Lake: Total Nitrogen

Fort Gibson Lake Total-Nitrogen			% R= 45% TMDL= 16,711.0 kg/day			
Source	Existing E(X) Mean	Existing % Share	LA kg/day	WLA kg/day	LA+WLA kg/day	Margin of Safety
Lake Hudson Inflow	5,012.2	68.0%	11,361.0	0.0	11,361.0	Implicit
Watershed HSPF	1,663.9	22.6%	3,771.5	0.0	3,771.5	Implicit
Small WWTP	29.0	0.4%	0.0	65.6	65.6	Implicit
Large WWTP	667.4	9.1%	0.0	1,512.9	1,512.9	Implicit
Total	**7,372.5**	**100.0%**	**15,132.5**	**1,578.5**	**16,711.0**	Implicit
NPDES Wastewater						
OK0043907 (S)	10.53	0.14%	0	23.9	23.9	Implicit
OKG380001 (S)	10.55	0.14%	0	23.9	23.9	Implicit
OK0033791 (S)	7.87	0.11%	0	17.8	17.8	Implicit
OK34568-006 (L)	570.46	7.74%	0	1,293.0	1,293.0	Implicit
OK0000272 (L)	17.12	0.23%	0	38.8	38.8	Implicit
OK0035149 (L)	80.22	1.09%	0	181.8	181.8	Implicit
Lake Hudson Inflow	Delta lognormal distribution					
Watershed HSPF	Lognormal distribution					
Small WWTP	Lognormal distribution					
Large WWTP	Lognormal distribution					

Table 13- Lognormal Parameters and Estimation of Maximum Daily Load for Total Organic Carbon for Watershed (HSPF) Runoff in 2006

Watershed Runoff to Lake Ft. Gibson: 2006 Lognormal Distribution Total Organic Carbon (TOC)			
Watershed Parameters, Log Transformed			
Ln (X, kg/day)		Arithmetic (X, kg/day)	
N (Obs)=	364		
μ=	7.331819	E(X)=	23794.43
Var=	5.490775	V(X)=	1.37E+11
StdDev=	2.34324	s(X)=	369729.7
CoeffVar=	0.319599	CV(X)=	15.5385
Min	0.94094	Min(X)=	2.56239
Max	12.45273	Max(X)=	255949.1
		1-sided, a=	0.05
		Probability,p=	0.95
		Arg (Ø)=	0.95
		Z(p)=	1.645
		R^2=	0.9886
Maximum Daily Load Parameters			
		X(p)=	72146.16
		VF(p)=X(p)/E(X)=	3.032061
		Existing Avg: E(X)=	23794.43
		% Removal: %R=	45%
		LTA= E(X*)*(1-%R)=	13086.93
		MDL= LTA*VF(p)=	39680.39

Table 14- Lognormal Parameters and Estimation of Maximum Daily Load for Total Organic Carbon for Small Wastewater Loads in 2006

Small Wastewater to Lake Ft. Gibson: 2006 Lognormal Distribution Total Organic Carbon (TOC)			
Small Wastewater Parameters, Log Transformed			
Ln (X, kg/day)		Arithmetic (X, kg/day)	
N (Obs)=	364		
μ=	3.604319	E(X)=	37.60606
Var=	0.045692	V(X)=	66.1177
StdDev=	0.213757	s(X)=	8.131279
CoeffVar=	0.059306	CV(X)=	0.216223
Min	2.994515	Min(X)=	19.97568
Max	4.002398	Max(X)=	54.72922
		1-sided, a=	0.05
		Probability,p=	0.95
		Arg (Ø)=	0.95
		Z(p)=	1.645
		R^2=	0.9856
Maximum Daily Load Parameters			
		X(p)=	52.2453
		VF(p)=X(p)/E(X)=	1.389279
		Existing Avg: E(X)=	37.60606
		% Removal: %R=	45%
		LTA= E(X*)*(1-%R)=	20.68333
		MDL= LTA*VF(p)=	28.73492

Table 15- Lognormal Parameters and Estimation of Maximum Daily Load for Total Organic Carbon for Large Wastewater Loads in 2006

Large Wastewater to Lake Ft. Gibson:2006 Lognormal Distribution Total Organic Carbon (TOC)			
Large Wastewater Parameters, Log Transformed			
Ln (X, kg/day)		Arithmetic (X, kg/day)	
N (Obs)=	364		
μ=	7.330719	E(X)=	1559.764
Var=	0.043141	V(X)=	107251.7
StdDev=	0.207703	s(X)=	327.4931
CoeffVar=	0.028333	CV(X)=	0.209963
Min	6.96718	Min(X)=	1061.226
Max	7.808403	Max(X)=	2461.197
		1-sided, a=	0.05
		Probability,p=	0.95
		Arg (Ø)=	0.95
		Z(p)=	1.645
		R^2=	0.9426
Maximum Daily Load Parameters			
		X(p)=	2148.211
		VF(p)=X(p)/E(X)=	1.377267
		Existing Avg: E(X)=	1559.764
		% Removal: %R=	45%
		LTA= E(X*)*(1-%R)=	857.8701
		MDL= LTA*VF(p)=	1181.516

Table 16- Delta Lognormal Parameters and Estimation of Maximum Daily Load for Total Organic Carbon from Lake Hudson inflow to Neosho River in 2006

Hudson Lake Inflow to Neosho River: 2006 Delta Lognormal Distribution Total Organic Carbon (TOC)			
Censored Parameters			
D =	2000	Min Load (kg/day)	
r=	286	Obs <= D	
N=	365	Total Obs	
ð=r/N	0.783562	Fraction censored data	
Non-Censored Parameters, Log Transformed			
Ln (X, kg/day)		Arithmetic (X, kg/day)	
k=N-r	79	Obs > D	
μ=	9.681703	E(X*)=	9211.795
Var=	1.581022	V(X*)=	1.23E+09
StdDev=	1.257387	s(X*)=	35078.69
CoeffVar=	0.129872	CV(X*)=	3.808019
Min	7.603807	Min(X*)=	2005.818
Max	11.6892	Max(X*)=	119276.1
		1-sided, a=	0.05
		Probability,p=	0.95
		Arg (Ø)=(p-ð)/(1-ð)	0.768987
		Z*p =	0.735516
		R^2=	0.9449
Maximum Daily Load Parameters			
		X*(p)=	40397.81
		VF(p)=X*(p)/E(X*)=	4.385444
		Existing Avg: E(X*)=	9211.795
		% Removal: %R=	45%
		LTA= E(X*)*(1-%R)=	5066.487
		MDL= LTA*VF(p)=	22218.8

Table 17 - Summary of Parameters and Maximum Daily Loads for External Sources of Total Organic Carbon to Fort Gibson Lake in 2006

Fort Gibson Lake: Jan-Dec 2006 Total Organic Carbon (TOC)					
Parameter	Lake Hudson Inflow	Watershed HSPF	Small WWTP	Large WWTP	Total
Min: D (kg/day)	2,000	0	0	0	
N-r (> D)	79	364	364	364	
r (< D)	286	0	0	0	
N	365	364	364	364	
E(X) (kg/day)	9,211.8	23,794.4	37.6	1,559.8	34,603.6
V(X)	1.2305E+09	1.3670E+11	66.1	107,251.7	
s(X) (kg/day)	35,078.7	369,729.7	8.1	327.5	
CV(X)	3.81	15.54	0.22	0.21	
Min(X) (kg/day)	2,005.8	2.6	20.0	1,061.2	
Max(X) (kg/day)	119,276.1	255,949.1	54.7	2,461.2	
Probability,p	0.950	0.950	0.950	0.950	
ð=r/N	0.784	0.000	0.000	0.000	
Arg (Ø)=(p-ð)/(1-ð)	0.769	0.950	0.950	0.950	
R^2=	0.945	0.989	0.986	0.943	
Z(p)	0.736	1.645	1.645	1.645	
X(p) (kg/day)	40,397.8	72,146.2	52.2	2,148.2	
VF(p)	4.39	3.03	1.39	1.38	
% Reduction	45%	45%	45%	45%	45%
LTA (kg/day)	5,066.5	13,086.9	20.7	857.9	19,032.0
MDL (kg/day)	22,218.8	39,680.4	28.7	1,181.5	63,109.4
Lake Hudson Inflow	Delta lognormal distribution				
Watershed HSPF	Lognormal distribution				
Small WWTP	Lognormal distribution				
Large WWTP	Lognormal distribution				

Table 18 - Maximum Daily Load Allocations for Fort Gibson Lake: Total Organic Carbon

Fort Gibson Lake Total Organic Carbon (TOC)			% R= 45% TMDL= 63,109.4 kg/day			
Source	Existing E(X) Mean	Existing % Share	LA kg/day	WLA kg/day	LA+WLA kg/day	Margin of Safety
Lake Hudson Lake	9,211.8	26.6%	16,800.3	0.0	16,800.3	Implicit
Watershed HSPF	23,794.4	68.8%	43,395.9	0.0	43,395.9	Implicit
Small WWTP	37.6	0.1%	0.0	68.6	68.6	Implicit
Large WWTP	1,559.8	4.5%	0.0	2,844.7	2,844.7	Implicit
Total	**34,603.6**	**100.0%**	**60,196.2**	**2,913.3**	**63,109.4**	Implicit
NPDES Wastewater						
OK0043907 (S)	30.2	0.09%	0.0	55.1	55.1	Implicit
OKG380001 (S)	4.3	0.01%	0.0	7.8	7.8	Implicit
OK0033791 (S)	3.2	0.01%	0.0	5.8	5.8	Implicit
OK34568-006 (L)	899.3	2.60%	0.0	1,640.1	1,640.1	Implicit
OK0000272 (L)	387.5	1.12%	0.0	706.8	706.8	Implicit
OK0035149 (L)	281.0	0.81%	0.0	512.5	512.5	Implicit
Lake Hudson Inflow	Delta lognormal distribution					
Watershed HSPF	Lognormal distribution					
Small WWTP	Lognormal distribution					
Large WWTP	Lognormal distribution					

Table 19- Delta Lognormal Parameters and Estimation of Maximum Daily Load for Total Suspended Solids for Watershed (HSPF) Runoff in 2006

Watershed Runoff to Lake Ft. Gibson: 2006 Delta Lognormal Distribution Total Suspended Solids (TSS)			
Censored Parameters			
D =	1	Min Load (kg/day)	
r=	96	Obs <= D	
N=	364	Total Obs	
ð=r/N	0.263736	Fraction censored data	
Watershed Parameters, Log Transformed			
Ln (X, kg/day)		Arithmetic (X, kg/day)	
k=N-r	268	Obs > D	
μ=	6.753958	E(X)=	147275.7
Var=	10.90454	V(X)=	1.6E+15
StdDev=	3.302202	s(X)=	40040648
CoeffVar=	0.488928	CV(X)=	271.8754
Min	0.061989	Min(X)=	1.063951
Max	15.31408	Max(X)=	4475281
		1-sided, a=	0.05
		Probability,p=	0.95
		Arg (Ø)=	0.93209
		Z(p)=	1.491536
		R^2=	0.992
Maximum Daily Load Parameters			
		X(p)=	118102.8
		VF(p)=X(p)/E(X)=	0.801916
		Existing Avg: E(X)=	147275.7
		% Removal: %R=	0.45
		LTA= E(X*)*(1-%R)=	81001.66
		MDL= LTA*VF(p)=	64956.52

Table 20- Lognormal Parameters and Estimation of Maximum Daily Load for Total Suspended Solids Organic Carbon for Small Wastewater Loads in 2006

Small Wastewater to Lake Ft. Gibson: 2006 Lognormal Distribution Total Suspended Solids (TSS)			
Small Wastewater Parameters, Log Transformed			
Ln (X, kg/day)		Arithmetic (X, kg/day)	
N (Obs)=	364		
µ=	2.319681	E(X)=	10.54983
Var=	0.072858	V(X)=	8.411735
StdDev=	0.269922	s(X)=	2.900299
CoeffVar=	0.116362	CV(X)=	0.274914
Min	1.693096	Min(X)=	5.436288
Max	3.032205	Max(X)=	20.74291
		1-sided, a=	0.05
		Probability,p=	0.95
		Arg (Ø)=	0.95
		Z(p)=	1.645
		R^2=	0.9736
Maximum Daily Load Parameters			
		X(p)=	15.85846
		VF(p)=X(p)/E(X)=	1.503195
		Existing Avg: E(X)=	10.54983
		% Removal: %R=	45%
		LTA= E(X*)*(1-%R)=	5.802409
		MDL= LTA*VF(p)=	8.722154

Table 21- Lognormal Parameters and Estimation of Maximum Daily Load for Total Suspended Solids for Large Wastewater Loads in 2006

Large Wastewater to Lake Ft. Gibson:2006 Lognormal Distribution Total Suspended Solids (TSS)			
Large Wastewater Parameters, Log Transformed			
Ln (X, kg/day)		Arithmetic (X, kg/day)	
N (Obs)=	364		
µ=	6.444129	E(X)=	642.1998
Var=	0.041541	V(X)=	17493.12
StdDev=	0.203816	s(X)=	132.2616
CoeffVar=	0.031628	CV(X)=	0.205951
Min	6.178387	Min(X)=	482.2136
Max	6.801411	Max(X)=	899.1151
		1-sided, a=	0.05
		Probability,p=	0.95
		Arg (Ø)=	0.95
		Z(p)=	1.645
		R^2=	0.8962
Maximum Daily Load Parameters			
		X(p)=	879.546
		VF(p)=X(p)/E(X)=	1.369583
		Existing Avg: E(X)=	642.1998
		% Removal: %R=	45%
		LTA= E(X*)*(1-%R)=	353.2099
		MDL= LTA*VF(p)=	483.7503

Table 22- Delta Lognormal Parameters and Estimation of Maximum Daily Load for Total Suspended Solids from Lake Hudson inflow to Neosho River in 2006

Hudson Lake Inflow to Neosho River: 2006 Delta Lognormal Distribution Total Suspended Solids (TSS)			
Censored Parameters			
D =	9000	Min Load (kg/day)	
r=	307	Obs <= D	
N=	365	Total Obs	
ð=r/N	0.841096	Fraction censored data	
Non-Censored Parameters, Log Transformed			
Ln (X, kg/day)		Arithmetic (X, kg/day)	
k=N-r	58	Obs > D	
µ=	10.78333	E(X*)=	27556.68
Var=	1.917895	V(X*)=	1.64E+10
StdDev=	1.384881	s(X*)=	128140.9
CoeffVar=	0.128428	CV(X*)=	4.650083
Min	9.124234	Min(X*)=	9174.968
Max	12.94351	Max(X*)=	418115
		1-sided, a=	0.05
		Probability,p=	0.95
		Arg (Ø)=(p-ð)/(1-ð)	0.685345
		Z*p =	0.482698
		R^2=	0.8662
Maximum Daily Load Parameters			
		X*(p)=	94071.83
		VF(p)=X*(p)/E(X*)=	3.413758
		Existing Avg: E(X*)=	27556.68
		% Removal: %R=	45%
		LTA= E(X*)*(1-%R)=	15156.17
		MDL= LTA*VF(p)=	51739.51

Table 23 - Summary of Parameters and Maximum Daily Loads for External Sources of Total Suspended Solids (TSS) to Fort Gibson Lake in 2006

Lake Fort Gibson: Jan-Dec 2006 Total Suspended Solids (TSS)					
Parameter	Lake Hudson Inflow	Watershed HSPF	Small WWTP	Large WWTP	Total
Min: D (kg/day)	9,000	1	0	0	
N-r (> D)	58	268	364	364	
r (< D)	307	96	0	0	
N	365	364	364	364	
E(X) (kg/day)	27,556.68	147,275.74	10.55	642.20	175,485.2
V(X)	1.6420E+10	1.6033E+15	8.41	17,493.12	
s(X) (kg/day)	128,140.86	40,040,647.89	2.90	132.26	
CV(X)	4.65	271.88	0.27	0.21	
Min(X) (kg/day)	9,174.97	1.06	5.44	482.21	
Max(X) (kg/day)	418,114.96	4,475,281.18	20.74	899.12	
Probability,p	0.95	0.95	0.95	0.95	
ð=r/N	0.841	0.263736264	0	0	
Arg (Ø)=(p-ð)/(1-ð)	0.685344828	0.932089552	0.95	0.95	
R^2=	0.8662	0.992	0.9736	0.8962	
Z(p)	0.483	1.491535738	1.645	1.645	
X(p) (kg/day)	94,071.83	118,102.76	15.86	879.55	
VF(p)	3.41	0.80	1.50	1.37	
% Reduction	45%	45%	45%	45%	45%
LTA (kg/day)	15,156.2	81,001.7	5.8	353.2	96,516.8
MDL (kg/day)	51,739.5	64,956.5	8.7	483.8	117,188.5
Lake Hudson Inflow	Delta lognormal distribution				
Watershed HSPF	Delta lognormal distribution				
Small WWTP	Lognormal distribution				
Large WWTP	Lognormal distribution				

Table 24 - Maximum Daily Load Allocations for Fort Gibson Lake: Total Suspended Solids

Fort Gibson Lake Total Suspended Solids (TSS)			% R= 45% TMDL= 117,188.5 kg/day			
Source	Existing E(X) Mean	Existing % Share	LA kg/day	WLA kg/day	LA+WLA kg/day	Margin of Safety
Lake Hudson Inflow	27,556.7	15.7%	18,402.3	0.0	18,402.3	Implicit
Watershed HSPF	147,275.7	83.9%	98,350.3	0.0	98,350.3	Implicit
Small WWTP	10.5	0.0%	0.0	7.0	7.0	Implicit
Large WWTP	642.2	0.4%	0.0	428.9	428.9	Implicit
Total	**175,485.2**	**100.0%**	**116,752.6**	**435.9**	**117,188.5**	Implicit
NPDES Wastewater						
OK0043907 (S)	5.3	0.00%	0	3.6	3.6	Implicit
OKG380001 (S)	2.6	0.00%	0	1.7	1.7	Implicit
OK0033791 (S)	2.7	0.00%	0	1.8	1.8	Implicit
OK34568-006 (L)	438.9	0.25%	0	293.1	293.1	Implicit
OK0000272 (L)	103.0	0.06%	0	68.8	68.8	Implicit
OK0035149 (L)	103.4	0.06%	0	69.1	69.1	Implicit
Lake Hudson Inflow	Delta lognormal distribution					
Watershed HSPF	Delta lognormal distribution					
Small WWTP	Lognormal distribution					
Large WWTP	Lognormal distribution					

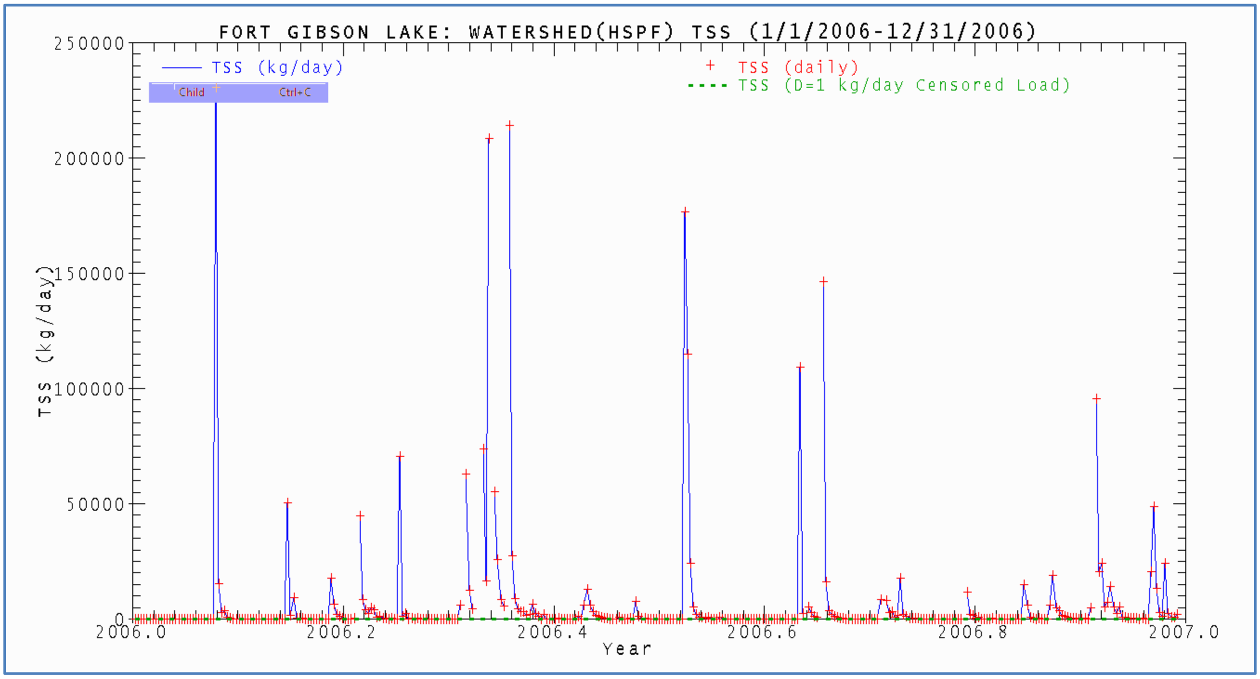

Figure 17 - Time series of TSS load from watershed runoff from HSPF model for 2006. Red markers and blue line show daily loading data from the HSPF model.

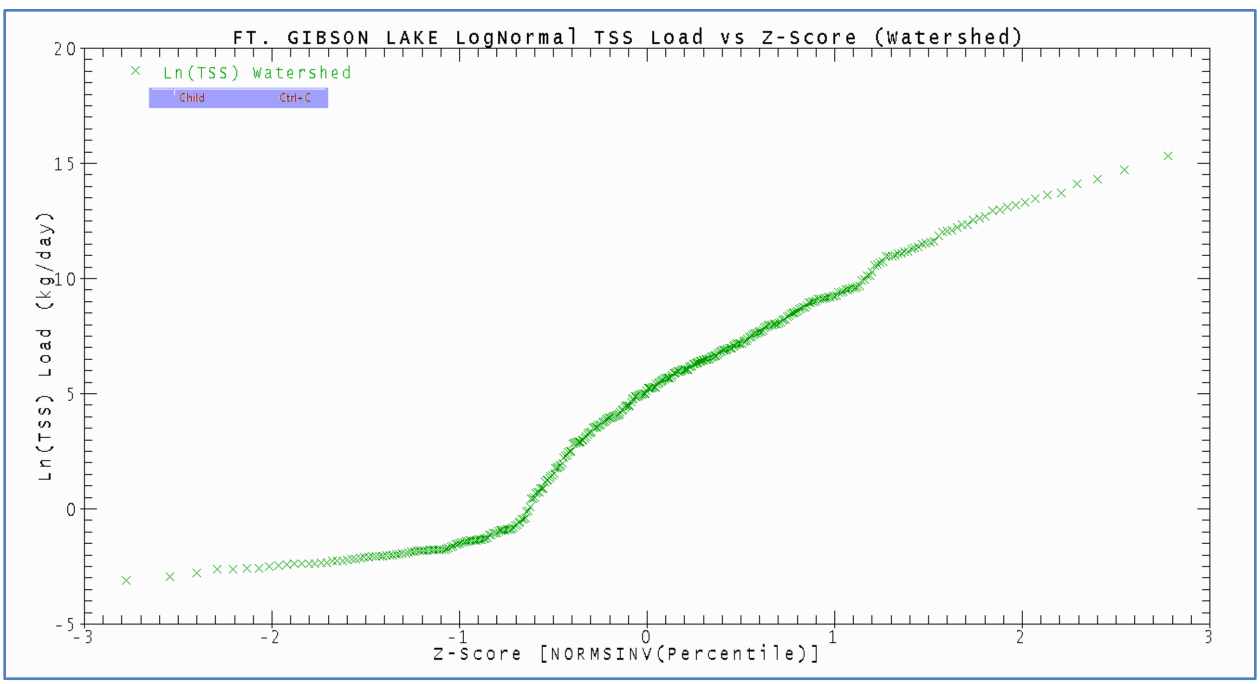

Figure 18 - Probability plot of lognormal distribution for watershed runoff from HSPF model for TSS load data from 2006 drought year (r^2=0.956)

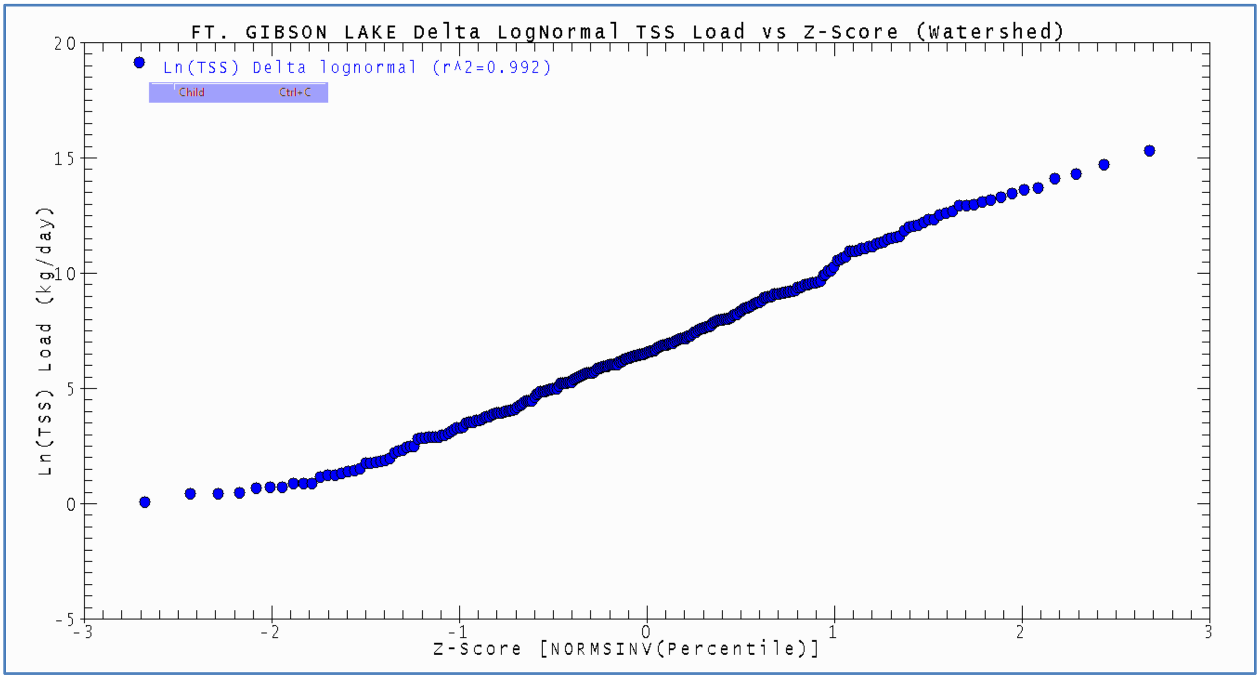

Figure 19 - Probability plot of delta lognormal distribution for watershed runoff from HSPF model for TSS load data from 2006 drought year. Censored TSS load is D=1.0 kg/day (r^2=0.992)

I-5 REFERENCES

Di Toro, D. M. (1984). Probability model of stream quality due to runoff. *J. Environ. Eng.*,110(3), :607–628.

EPA (1984). Technical Guidance Manual for Performing Wasteload Allocations, Book VII: Permit Averaging Periods. Office of Water, EPA-440/4-87.002.

EPA (1987). Development Document for Effluent Guidelines and Standards for the Organic Chemicals, Plastics and Synthetic Fibers Point Source Category. Volume I and Volume II, EPA 440/1-87/009, Washington, DC.

EPA (1991). Technical Support Document for Water Quality Based Toxics Control, EPA-505/2-90-001, Washington, DC.

EPA (2002). Development Document for Final Effluent Guidelines and Standards for the Iron and Steel Point Manufacturing Source Category. EPA-821-R-02-004, Washington DC.

EPA (2007). Options for Expressing Daily Loads in TMDLs. Office of Wetlands, Oceans & Watersheds, U.S. Environmental Protection Agency, Washington DC., Draft Report.

Grumbles, B.H. (2006). Establishing TMDL "Daily" Loads in Light of the Decision by the U.S. Court of Appeals for the D.C. Circuit in Friends of the Earth, Inc. v. EPA, et al., No. 05-5015, (April 25, 2006) and Implications, for NPDES Permits. Memorandum, November 15, 2006. https://www.epa.gov/tmdl/establishing-tmdl-daily-loads-light-decision-us-court-appeals-dc-circuit (updated sept-30-2021)

Kahn, H.D. and M. B. Rubin. (1989). Use of statistical methods in industrial water pollution control regulations in the United States. Environmental Monitoring and Assessment, 12: 129-148.

Limbrunner, J. F., Vogel, R. M., and Brown, L. C. (2000). Estimation of harmonic mean of lognormal variable. *J. Hydrologic Eng.*, 5(1), 59–66.

Nash, D. B. (1994). Effective sediment-transporting discharge from magnitude-frequency analysis. *J. Geol.*, 102, 79–95.

Owen, W. and T. DeRoen. (1980). Estimation of the mean for lognormal data containing zeroes and left-censored values with applications to the measurement of worker exposure to air contaminants. Biometrics, 36, 707-719.

Sandbo, B., et al. (2008). Weathering Drought: One State's Experience. P1.29, 3rd Symposium on Policy and Socio-Economic Research, American Meteorological Society, New Orleans, LA.

Tortorelli, R.L. (2008). Hydrologic Drought of Water Year 2006 Compared with Four Major Drought Periods of the 20th Century in Oklahoma. USGS Scientific Investigations Report 2008-5199.

Van Buren, M A., Watt, W. E., and Marsalek, J. (1997). Application of the lognormal and normal distributions to stormwater quality parameters. *Water Res.*, 31(1), 95–104.

Vogel, R.M., Rudolph, B., and R.P. Hooper, (2005). The Probabilistic Behavior of Water Quality Loads, *Journal of Environmental Engineering*, 131 (7): 1081-1089.