APPENDIX 2
MULTILAYER PRESSURE MODEL

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1.0 MULTILAYER PRESSURE MODEL - GENERAL DESCRIPTION

This model has been developed to calculate the pressure increase produced within various permeable layers of an underground geological system as a result of waste injection. It is an extension of an earlier pressure model described in detail by Miller et al. (1986).

The earlier development considered only a single permeable injection layer bounded above and below by totally impermeable strata. The model was based on the well-known Theis equation (1935), originally formulated for a single well injecting at a constant rate. By making appropriate use of time and spatial superposition principles, the model was extended to account for multiple injection wells operating at variable injection rates (Thornhill et al., 1982).

The new development further improves the earlier model (Miller et al., 1986) by providing a more detailed and refined picture of system behavior. It recognizes the multiple layer nature of many geological environments, and the potential ability of the various permeable strata of such systems to communicate with one another in terms of both pressure and fluid flow.

At a well that is completed into more than one reservoir at a given time, the various perforated reservoirs can communicate with one another via the wellbore itself (Figure 1). Such an arrangement is referred to in the petroleum industry as a commingled or stratified reservoir (Raghaven, 1986; Shah et al., 1986). Considerable work has been done in quantifying the parameters that influence system response for simplified arrangements of commingled reservoirs (Raghaven, 1986).

Communication also may occur by slow vertical permeation of fluids through the low-permeability aquitards which separate the injection interval and other high-permeability layers from one another (Figure 2). The permeation rate through each aquitard is assumed to be proportional to the pressure differential across it. This discounts the compressive storage capacity of the aquitard, and is a conservative assumption, as discussed in further detail below. Models that are capable of analyzing slow vertical permeation through low-permeability aquitards are referred to in the petroleum industry as crossflow models (Raghaven, 1986).

2.0 MODEL UTILIZATION

There are two primary applications for the Multilayer Pressure Model. The first is in predicting the pressure distribution within the injection interval as a function of time and position, for use in Area of Review evaluations. The second is in predicting the time-dependent variations of pressure buildup at the injection wells. This pressure buildup is subsequently employed as an input to the Multilayer Vertical Permeation Model to predict the extent of waste permeation into the overlying aquitard both during and following the injection period. The Multilayer Pressure Model also provides certain input information to the Basic Plume Model which calculates the growth of the waste plume emerging from the wells at an injection site. For sites with at least one well perforated into more than one injection interval at a given time, the Multilayer Pressure Model calculates partitioning of the injection flow between the available intervals. It also determines interformational flow taking place at wells that are not injecting waste, but which are completed into more than one horizon.

Another application of the Multilayer Pressure Model is the iterative process of calibration between model-generated and observed pressure variations. This process includes determining transmissivities of the high permeability layers, as well as placing loose bounds on the aquitard layer permeabilities. The calibration process involves pressure history matching against observed bottom-hole pressure over the past operating life of the site. The calibrated parameter values thus obtained provide an increased degree of confidence for use in analysis of Area of Review and vertical fluid permeation calculations.

3.0 MODEL STRUCTURE

The model is structured as a general purpose generic tool, capable of being applied to a wide variety of site-specific geological arrangements. It consists of a series of alternating high- and low-permeability layers, numbered in sequence from the top of the layer stack to the bottom (Figure 3). The high-permeability units are assigned even numbers in the sequence, while the low permeability aquitard layers are assigned odd numbers. Each layer corresponds to an identifiable stratigraphic unit at the site.

4.0 MODEL INPUTS

Model inputs refer to data that must be supplied to the computer program to perform a site-specific calculation, together with the potential sources of these data. Inputs to the model are given in the form of fundamental physical properties for the geological strata and fluids. These differ from the reservoir properties used as input to some models, which are specially defined combinations of the fundamental properties.

4.1 Well Locations

The geographical coordinates of the well are specified in terms of an X-Y coordinate system inscribed onto a map of the site locale. Data on the location of the well are readily available from historical records and maps of the plant vicinity. For a deviated wellbore, the borehole is projected downward to its intersection with the injection horizon.

4.2 Geological Data

4.2.1 Average Thickness of Layers

The average thicknesses of the geological units in the model are determined by analyzing resistivity and spontaneous potential logs. Special geological surveys by experienced consulting organizations are often used as additional data sources.

Predicted pressure buildup within an injection interval typically will vary in inverse proportion to the thickness of the injection interval. Thicknesses of the aquitard layers have only a very slight influence on the pressure response of an underground injection system, as evidenced by the ability of single-layer models which do not even include the aquitard layers to predict the pressure response.

4.2.2 High-permeability Layer Properties

4.2.2.1 Average Horizontal Permeability

The average horizontal permeability of these layers can be determined from conventional and sidewall core samples, injectivity/falloff tests, interwell interference tests, or may be estimated from regional geological studies if no site data exist. A large number of core sample data is desirable to obtain a representative average. The core-derived permeability values can be improved by calibrating the model to historic data at shut-in wells and injecting wells.

Horizontal permeabilities of the strata used for injection constitute first-order parameters in determining the pressure response of the system. Predicted pressure buildup within each injection interval will vary in nearly inverse proportion to the permeability of that layer. For relatively permeable layers not used for injection, model results are quite insensitive to the choice of permeability, since a significant amount of fluid must first pass through the aquitards before the effects of these layers are felt.

4.2.2.2 Average Layer Porosity

Average layer porosities are determined from core samples, geophysical porosity logs, or regional geological studies of the injection interval. Porosity enters into the model only through the contribution of fluid compressibility to the overall layer storativities. Storativity is a reservoir parameter which expresses the combined effects of layer porosity and compressibility. Model results are quite insensitive to the layer storativities, and therefore, also to the porosity values used. Typically, a 10 percent change in porosity will result in less than a 0.5 percent change in predicted pressure buildup.

4.2.2.3 Rock Compressibility

Compressibilities of strata are rarely measured for the geological layers at an underground injection site. Input values for these parameters can be estimated using information from the literature (Freeze and Cherry, 1979) for similar strata. In the model, compressibilities enter into the calculation only by combining with a fluid compressibility term in determining the layer storativities. Since, as mentioned above, model results are very insensitive to layer storativities, they also will be quite insensitive to the values chosen for the compressibilities. The predicted pressure buildup will typically change by less than 10 percent for a factor of 10 change in compressibility.

4.2.3 Low-permeability Layer Properties

Properties required by the model for aquitard (low permeability) layers are vertical permeability and porosity. Actually, porosity is not used in the Multilayer Pressure Model calculation, but it is employed in the Vertical Permeation Model which adopts the same input format. Therefore, in supplying the Multilayer Pressure Model with these input data, these data are actually stored for later use by the Vertical Permeation Model.

Values of aquitard permeability and porosity rarely have been measured at underground injection sites. It would be unlikely to find data on these parameters, however, their measurement is becoming more frequent under new well installation requirements. Without site-specific measurements, it is necessary to make reasonably conservative estimates based on general information in the literature on similar aquitard materials (Freeze and Cherry, 1979; Neuzil, 1986) or from information for the same geological region.

Predicted pressure buildups are found to be very insensitive to the values selected for aquitard permeabilities, since these permeabilities are generally very low and do not allow for much fluid bleed-off from the injection interval into the aquitards. The ability to calibrate the results of the Multilayer Pressure Model against historic site-specific operating data suggests that the actual aquitard permeability values do not grossly exceed estimates from regional data.

4.3 Well Radius

The model requires data specifying the radius of each active well at an underground injection site. The well radius can be determined from caliper logs for the well. The model results are virtually independent of these parameters at all locations other than very near the wells. Even directly at the wells, the results are very insensitive to the values specified for the radii. Typically, a 10 percent change in well radius will result in less than a one percent change in the predicted pressure buildup.

4.4 Fluid Viscosity

A separate fluid viscosity value is specified for each layer in the model. If fluid samples were taken during drilling, the viscosity can be measured directly with appropriate corrections made for the temperature at depth. When no fluid samples are available, viscosity can be estimated accurately from property data in the literature (Lobo, 1984; Suryanarayana and Venkatesan, 1958; Perry and Chilton, 1973), which give saline brine viscosity as a function of temperature and salinity.

After the model has been calibrated against site-specific operating data, the value of the transmissivity for each of the injection layers in the model will be known more accurately. Since transmissivity is proportional to the ratio of permeability to viscosity, this ratio will be better characterized. It is actually the ratio, and not individual parameters, that determines the pressure response in the model. Therefore, even if the individual parameters are not known very accurately, their ratio can be calibrated and used to accurately predict pressure buildup.

4.5 Site Operating History

The history of the injection rates into each well and the history of well completions into each injection interval must be specified. This information is available from site operating records. The information can be supplied to the model on an average annual basis, or, preferably, on an average monthly basis.

5.0 MODEL OUTPUTS

5.1 Reservoir Properties

Reservoir parameters are special mathematical combinations of the fundamental physical properties of geological strata and fluids. They include the permeable layer transmissivities and storativities and the aquitard leakances. It is these parameters alone which appear as the coefficients in the hydrological flow equations and govern the pressure response in the model. Therefore, in calibrating the model to site-specific operating data, it is appropriate to calibrate with respect to the reservoir properties rather than the fundamental physical input parameters.

5.2 Injection Rate at Each Well into Each Interval Over Time

If wells are operated such that only one horizon is perforated at any one time, flow into the injection interval will be identical to the injection rate at the wellhead. For a well perforated into more than one horizon at a time, the total injection flow rate at the wellhead is known but typically not the partitioning of the flow between the receiving horizons (unless flow meter logs have been frequently run). This partitioning is calculated in the Multilayer Pressure Model, together with the interformational flow taking place at wells not injecting waste but which are completed into more than one horizon. These calculated injection rates into the individual layers are subsequently employed as inputs to the Multilayer Vertical Permeation Model and the Basic Plume Model.

5.3 Pressure Buildup as a Function of Location and Time

The spatial distribution of pressure buildup with time is the primary output from the Multilayer Pressure Model. It can be displayed graphically by plotting pressure buildup as a function of time at a given location, or on a contour plot, showing lines of constant pressure (isopleths) at any given time. Contour plots are very useful in evaluating Area of Review, since they can include the locations of all boreholes in the region of the injection site.

6.0 ASSUMPTIONS, VALIDATION, AND MARGINS OF SAFETY

This section describes the key assumptions in the model, and evaluates particular site-specific conditions under which the assumptions are valid. Also summarized are various margins of safety built into the Multilayer Pressure Model to guarantee a conservative, upper-bound prediction of the pressure buildup within an underground injection system.

6.1 Flow is Horizontal in the High-permeability Layers, and Vertical in the Low-permeability Layers

According to Neuman and Witherspoon (1969a, b), this assumption is valid whenever the permeability of the high permeability layer is at least 100 times greater than that of the adjacent aquitards (low permeability layers). This condition is virtually always satisfied at underground injection sites.

6.2 Properties and Thicknesses of the Layers do not Vary with Position

Miller et al, (1986) and Collins (1986) have discussed the insensitivity of the pressure distribution developed as a result of injection to vertical variations in hydrogeologic properties (primarily permeability) through the thickness of the injection interval. They have indicated that pressure response is determined mainly by the thickness weighted average of layer properties. The ability of classical "Reservoir Analysis" to predict pressure buildup and drawdown in geological formations known to be highly nonuniform in the vertical direction is direct evidence of this insensitivity.

Properties of the layers, as well as their thicknesses, also can vary in the horizontal direction. The pressure response of an injection interval is determined by the product of the permeability and layer thickness, expressed as the transmissivity. The model includes the capability of handling large, sudden lateral variations in transmissivity, such as a fault blockage, using the method of image wells (Freeze and Cherry, 1979). Effects of gradual changes, such as pinch-outs, can be simulated by modeling these changes as sudden variations imposed at an appropriate location.

6.3 Density of the Waste is the Same as that of the Formation Fluid

The effect on pressure distribution of a density difference between the waste and formation fluid has been analyzed by Miller et al. (1986). They have shown that for a single injection interval, the inaccuracy in discounting this effect approaches zero in the center of the injection interval and is extremely small elsewhere within the waste plume, amounting to no more than half the product of the density difference (gm/cm^3) and injection interval height (feet), where the inaccuracy is expressed in feet of head. Outside the waste plume, the effect decreases very rapidly with distance away from the well. Density therefore has virtually no bearing on the Area of Review evaluation.

6.4 Viscosities of the Waste and Formation Fluid are Equal

In the high-permeability injection interval, the fluid viscosity varies from that of the waste in the region near the well to that of the formation fluid viscosity at distances further out. Since the injected waste temperature is normally lower than the temperature of the formation fluid, its viscosity may be expected to be slightly higher. This viscosity variation typically will amount to no more than a factor of two. The effect of assuming that the waste viscosity is equal to the formation fluid viscosity will be relevant only within the borders of the waste plume. Outside the plume, the predicted pressure distribution will be virtually unaffected.

If the value of the injection interval permeability is calibrated to pressure data at shut-in wells contained within the plume, the permeability value so determined will be underestimated. Use of this underestimated permeability value in predicting the pressure distribution throughout the injection interval will lead to an overestimate of the pressures at locations within the Area of Review, such as at abandoned boreholes. Thus, the assumption that the waste viscosity is the same as the formation fluid viscosity provides an added margin of safety in the Area of Review calculations.

In predicting vertical waste permeation into the overlying aquitard, use of the formation brine viscosity in the calculation will result in a slight underestimate of the permeation distance. However, the extent of this underestimate typically will be quite minimal compared to the uncertainty associated with the choice of the aquitard permeability value input to the Multilayer Vertical Permeation Model. This choice is normally very conservative by several orders of magnitude. Therefore, the inaccuracy associated with the viscosity assumption can be absorbed into the margin of safety factor for aquitard permeability.

6.5 Compressive Storage in the Low-Permeability Layers is Negligible

The ability of aquitard layers to compressively store fluids is assumed to be negligible in the Multilayer Pressure Model. Under this assumption, the pressure profile within the aquitards will reach an instantaneous steady state, varying linearly with vertical position and determined by pressure values at the boundaries with the two permeable layers immediately underlying and overlying the aquitard. Moreover, fluid permeation rates across the top and bottom boundaries of the aquitard will be equal and proportional to the pressure difference between the underlying and overlying permeable layers.

Use of this assumption consistently will lead to an overestimate of the pressure within the injection interval during injection. This is because compressive storage of fluids in the aquitards would provide an added sink for bleed-off of fluids and pressure from the injection interval. Therefore, discounting storage provides an added margin of safety in terms of using the Multilayer Pressure Model to calculate both Area of Review and vertical waste permeation into the overlying aquitard.

6.6 The Injection Interval is Fully Perforated

Not all injection wells are perforated over the full height of the injection interval. Effects of partial perforation are important only in the region relatively near the injection well, up to radial distances on the order of a few thicknesses of the injection interval, at most. The actual radial region of influence depends on the fraction of the height perforated and, to a minor extent, on the ratio of vertical to horizontal permeability. Within this region, the pressure buildup will vary both with vertical position through the formation as well as radial position away from the well.

The nature of this variation was studied extensively by Hantush (1964), who provided analytical relationships for quantifying the behavior. Hantush (1964) found that for constant injection rate after a short initial transient period, the contribution of partial perforation effects to the pressure distribution will approach steady state.

Hantush's steady-state equations (Hantush, 1964) have been employed to assess the influence of partial perforation on the pressures at an injection well and at the base of the overlying aquitard. In general, at the well, the effects will be analogous to those associated with well inefficiency or "skin effect." They will increase the pressure in the wellbore during injection in proportion to the injection rate and will have essentially no influence on shut-in pressures (after the initial transients have died out). Since the Multilayer Pressure Model is normally calibrated to shut-in pressures, the calibration results will not be affected by partial perforation. In cases where the model is calibrated to flowing bottomhole pressures, these calculated flowing pressures are corrected for skin and partial penetration (spherical flow) effects. Furthermore, since the effects of partial perforation are important only in the region near the wells, the Area of Review results also will be unaffected.

The influence of partial well perforation on the pressure at the base of the overlying aquitard is important in estimating vertical permeation. In general, Hantush's results (Hantush, 1964) have shown that, if the perforations are located in the lower portion of the injection interval, as is often the case, the pressure at the base of the overlying aquitard actually will be lower than if the full interval were perforated. Therefore, discounting the partial perforation effect will result in an overestimate of the pressure at the base of the overlying aquitard, and will thus provide an added margin of safety in calculating vertical permeation.

7.0 FORMULATION AND SOLUTION

Based on fundamental freshwater the assumptions discussed in the previous section, the partial differential equation describing the buildup of head (h_i)(which is proportional to the pressure buildup) within each permeable layer i of an underground hydrological system as a function of time and radial distance from an isolated injection well(s) given by Hunt, 1985:

$$S_i \frac{\partial h_i}{\partial t} = T_i \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial h_i}{\partial r} \right) + R_{i+1}\left(h_{i+2} - h_i \right) + R_{i-1}\left(h_{i-2} - h_i \right) \qquad (1)$$

where T_i and S_i are the transmissivity and storativity of permeable layer i, and R_{i+1} and R_{i-1} are the leakances for the overlying and underlying aquitards. These parameters are defined as follows:

$$T_i = \frac{\rho g k_i \; b_i}{\mu_i}$$

$$S_i = \rho g b_i \left(\alpha_i + \phi_i \beta_i \right)$$

$$R_i = \left(\frac{\rho g k_i}{\mu_i \; b_i} \right)$$

Where:

ρ	=	Density of fresh water,
g	=	Gravitational acceleration constant,
k_i	=	Permeability of Layer i,
μ_i	=	Fluid viscosity in Layer i,
b_i	=	Thickness of Layer i.
α_i	=	Compressibility of Layer i,
ϕ_i	=	Porosity of Layer i,
β_i	=	Compressibility of water.

The horizontal boundary conditions on the flow are given by

$$h_i = 0 \, at \; r \rightarrow \infty \qquad (2)$$

And

$$2\,\pi r\,T_i\,\frac{\partial h_i}{\partial r} = -\,Q_i\ \ at\ r = r_o \tag{3}$$

Where r_o is the well radius and Q_i is the volumetric injection rate into Layer i. Equation 2 implies that the layers are infinite in lateral extent. However, bounded reservoir conditions, and other lateral blockages, can be included by superposition techniques using the method of image wells.

In the vertical direction, no flow is permitted across the lower boundary of the model (downward out of the lowest layer in the stack), while the upper boundary is treated as a zero-head-buildup layer. The location of this boundary typically is taken as the depth of the lowest underground source of drinking water (USDW). This is very conservative from the standpoint of estimating vertical permeation effects, since the actual location of the zero-head-buildup boundary is at the ground surface (water table); thus, the model takes no credit for the vertical flow resistance provided by the aquitard layers overlying the lowermost USDW. In the Multilayer Pressure Model, the choice of the USDW as the zero-head-buildup boundary does not significantly influence the accuracy of the calculations, since this location typically is separated from the injection zone by several intervening aquitard layers, which effectively prevent the pressure variations from propagating to the USDW depth during the operating lifetime of a site.

For a multiple-well site injecting waste at variable rates into wells that are perforated into more than one interval simultaneously, the model makes use of well-established spatial- and time-superposition principles (Freeze and Cherry, 1979) to determine the head buildup. These superposition principles take advantage of the linearity of the basic partial differential equations and boundary conditions with respect to head buildup, together with the assumed spatial independence of layer properties, to calculate the solution for more complex situations by adding together the results for simpler spatial and time arrangements.

The first step in the solution procedure is to generate a set of time-dependent solutions to the response of a single isolated well injecting at a constant unit injection rate, into a specified injection interval. A separate solution is generated for each permeable interval used for injection. This involves solving the basic partial differential equation and boundary conditions given above numerically, and storing the results of the calculations in a set of tables of head buildup as a function of time, radial position, permeable layer index, and injection interval index.

The equations are solved by discretizing the partial differential equations with respect to the radial coordinate using a finite difference approach and solving the resulting set of ordinary differential equations (ODEs) in time by means of a commercially available stiff ODE package. The discretization scheme is carried out in the logarithm of the radius, rather than the radius itself, to achieve an added degree of accuracy, since the head buildup is approximately linear in log(r). The boundary condition at large radial distances from the well (zero-head-buildup) is applied at the final grid point, which is typically placed at a radius of about 200 miles.

Tables of solutions generated for the case of a single isolated well injecting at a constant unit injection rate into a specified injection interval are used to construct the solutions for the actual injection conditions of variable pumping rates and multiple-wells, by applying the principles of spatial- and time-superposition. The only complicated step in this process is accounting for wells that are perforated into several intervals simultaneously. The total injection rates into these wells are known in advance, but typically not the apportionment of the total flow among the available injection intervals. In the Multilayer Pressure Model, this apportionment is calculated by applying an additional boundary condition at all wells completed into more than one horizon. This boundary condition is that the head buildups for all perforated intervals at a given well location are the same.

The physical justification for this requirement is that the vertical flow resistance within the injection tubing is typically very low, so that head loss resulting from downward flow through the section of tubing connecting perforated intervals will be negligible. The well essentially acts as a high conductivity vertical pathway. Implementation of this boundary condition in the model requires the solution of a set of simultaneous linear equations for the flow rates into the individual injection intervals at each increment of time.

8.0 VERIFICATION

The computer model has been verified against a variety of solutions in the literature and with numerical tests to guarantee that it delivers reliable solutions to the model description as stated. In tests involving an isolated well injecting at constant rate into a single interval with impermeable aquitards, the model duplicated the results of the Theis (1935) equation to within 0.2 percent. Similar accuracy was obtained in comparisons against the Hantush and Jacob (1955) steady-state equation, which assumes the existence of a low permeability aquitard between the injection interval and the zero-head-buildup boundary.

A more comprehensive test of the single-well capabilities of the model was provided by a comparison with the results of Hunt (1985). Hunt solved for the head distribution in a multilayer stratigraphic system for the same set of equations as in the present model, but used an analytic solution technique. The geological arrangement in Hunt's example is shown in Figure 4, and the specific parameter values chosen are given in Table 1. Figure 5 compares the steady-state results from the present model with those of Hunt, while Figure 6 presents the corresponding comparison of the time-dependent behavior at a selected location. The two sets of results are identical, to within plotting accuracy. It is interesting to observe that El Didy and Contractor (1987) have performed this same intercomparison with Hunt's calculations to validate their two-dimensional finite element model of a multilayer system (El Didy, 1986).

The above tests fully verify the ability of the model to generate the response of a single isolated well injecting at constant rate into an individual interval, including the numerical discretization scheme employed for the radial coordinate.

The variable injection rate (time-superposition) aspect of the present model was verified by the following procedure. The radially discretized ordinary differential equations describing the time-dependent response of an isolated well injecting into a specified interval can be integrated numerically, not merely for the usual case of a constant injection rate (which is used to generate the tabulated well influence functions used in the model), but also for the case of fully variable injection rate history. On this basis, an integration was performed for a series of step changes in injection rate, and the results of the calculation were compared with output obtained directly from the model using the time-superposition software. Results from the two calculations were identical.

The multiple-well superposition feature of the model was checked using hand calculations for the case of two injection wells located at various coordinate positions injecting at constant rate into a formation bounded above and below by impermeable aquitards. The single-well response for this situation is governed by the Theis equation.

Results from the model also were compared to the analytic solution for the steady-state head buildup distribution produced by a combination of an injection well and an adjacent withdrawal well, both operating at the same constant flow rate. This solution is given in Bear (1972). The model output agreed with the analytic results to considerably better than 0.1 percent.

It also has been verified that at a well perforated into multiple horizons, the calculated apportionment of injection flow to the individual receiving horizons is consistently determined correctly. This was accomplished by confirming, in all instances, that the head-buildups within the various perforated intervals were identical in agreement with the required boundary condition, and that the sum of the flow rates to the perforated intervals precisely matched the total injection rate at the wellhead.

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Table 1
Parameters Used In Hunt's Multi-aquifer Example

Aquifer	Injection Rate (m^3/s)	Transmissivity (m^2/s)	Storativity	Overlying Aquifer Leakance (s^{-1})
1	0	0.0004	0.00005	0
2	0	0.0009	0.0004	0.000003
3	0	0.0007	0.00008	0.000008
4	0.04	0.003	0.0002	0.00001
5	0	0.001	0.0001	0.00003

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FIGURES

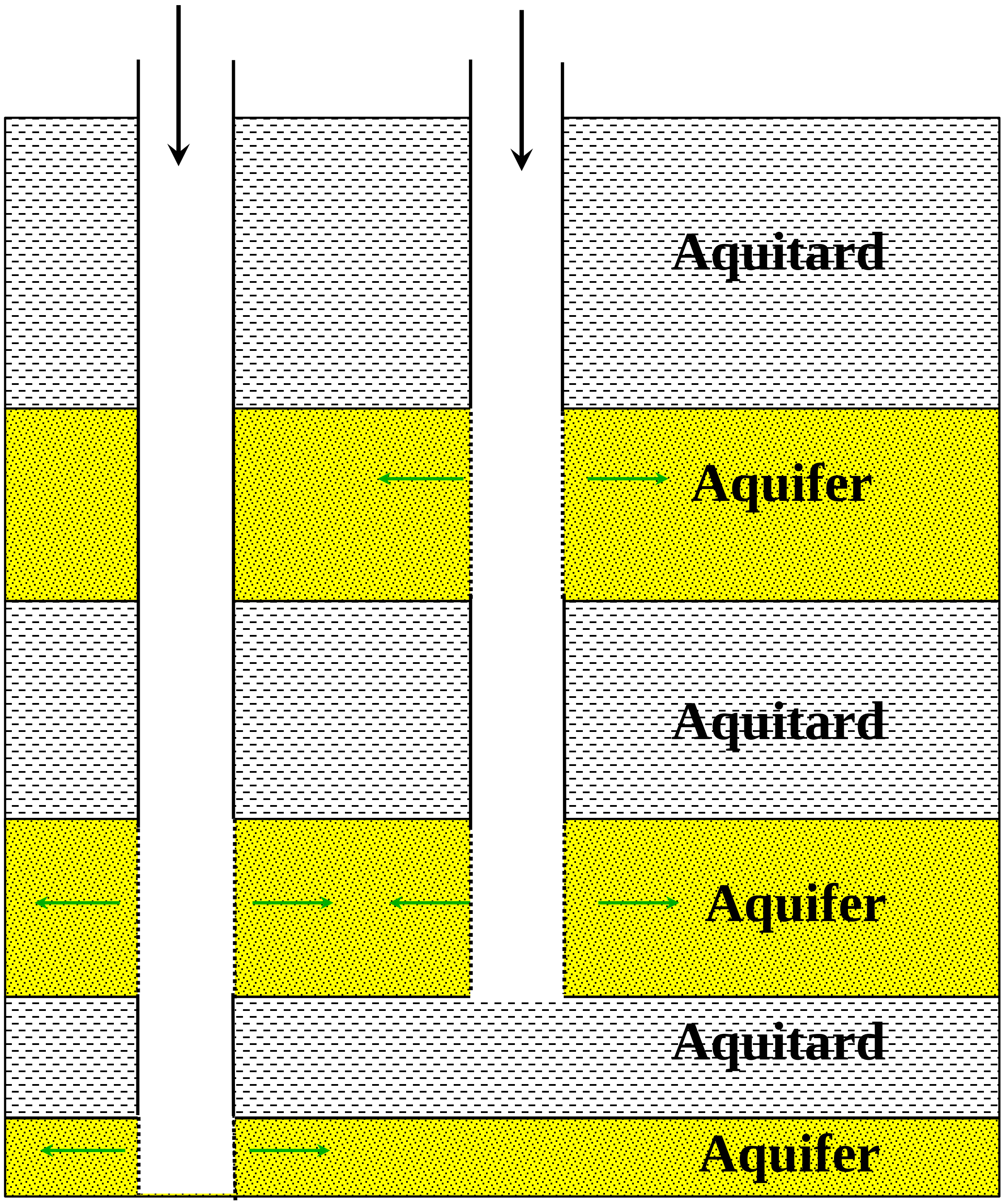


Figure 1
Injection Wells Perforated into More than One Horizon (Commingled)

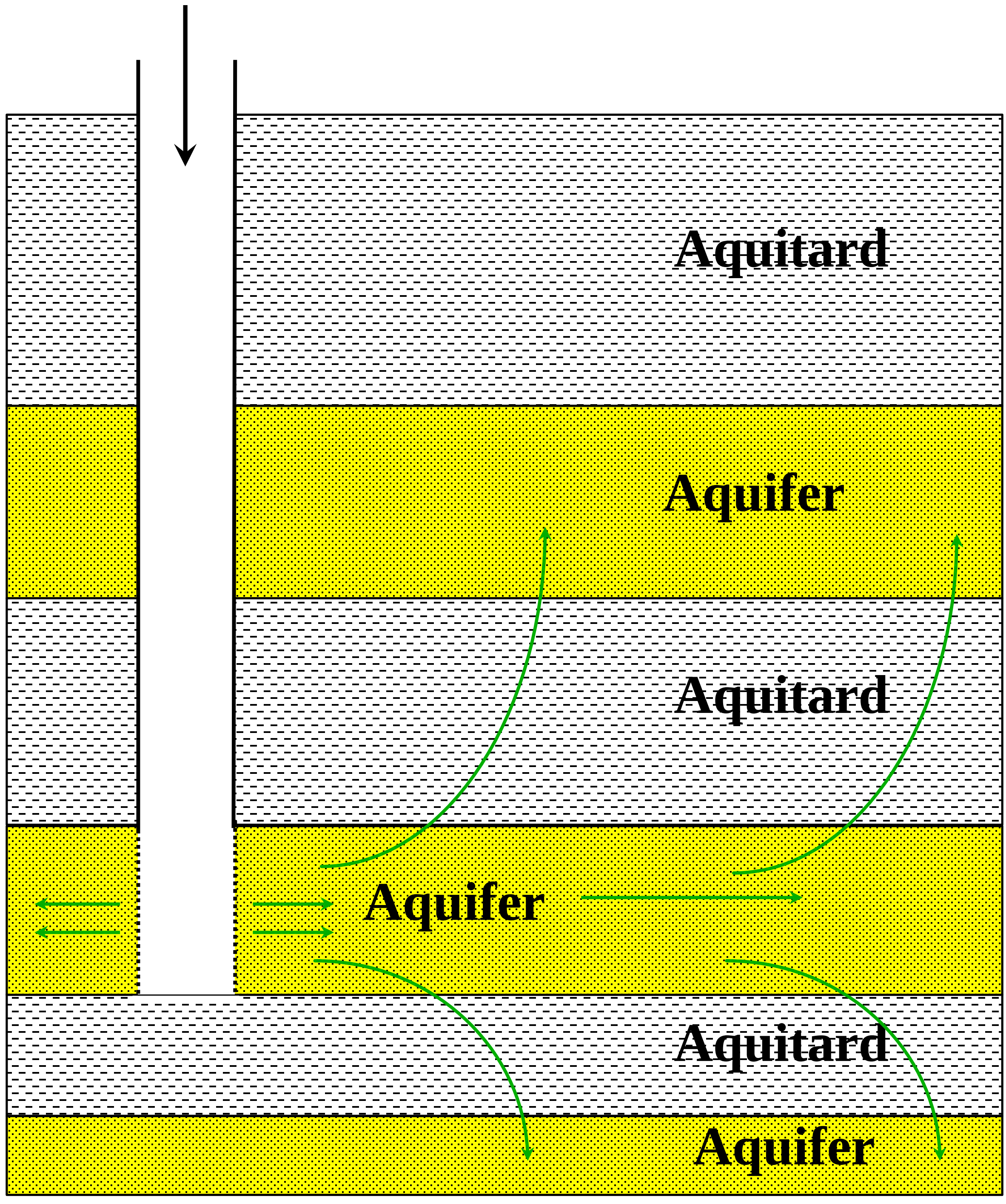
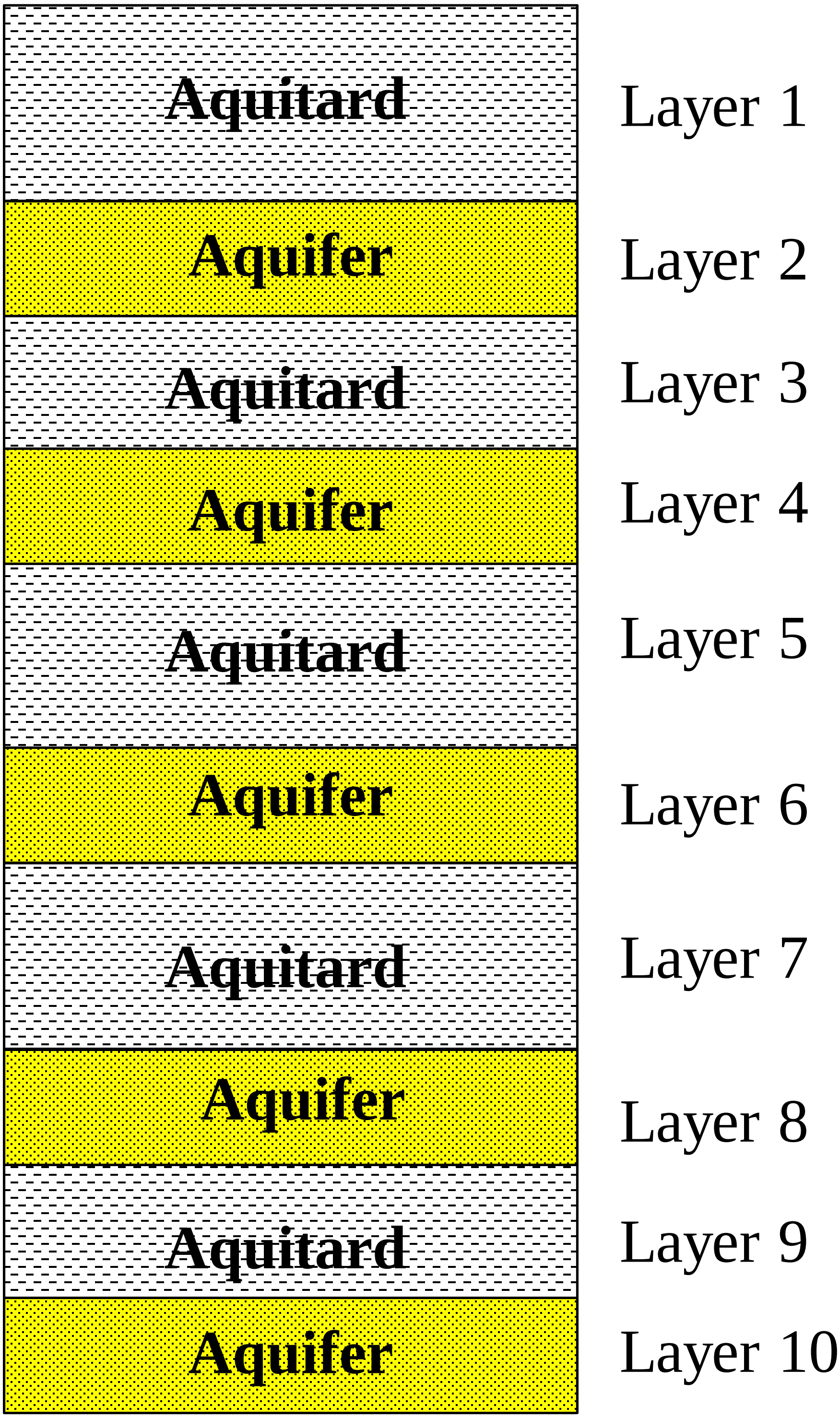


Figure 2
Fluid Seepage through Aquitards (Crossflow)

Geologic Column of Layers



Note : Multilayer Pressure Model must end with an aquifer layer

Figure 3
Model Structure for Typical Geological Arrangement

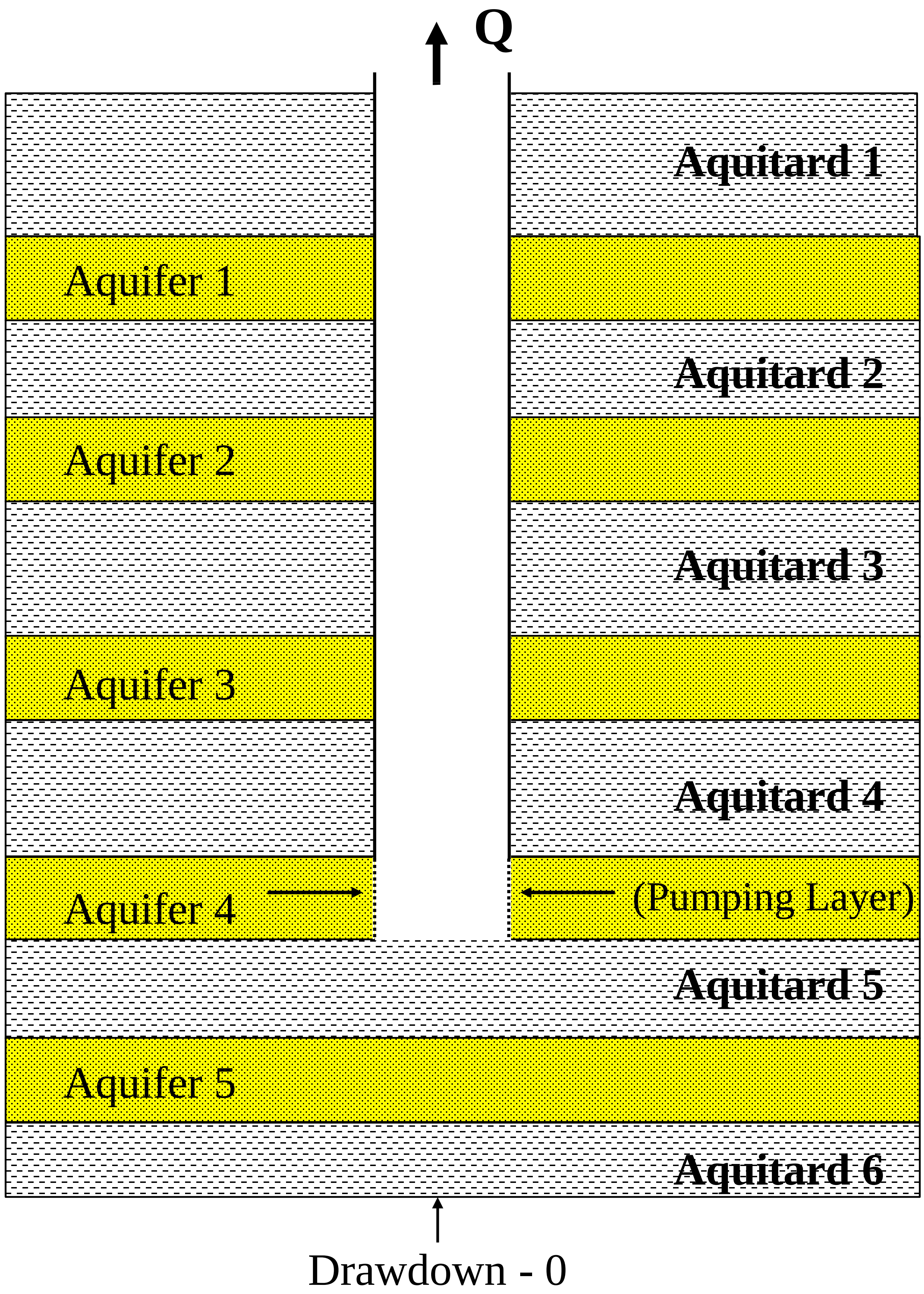


Figure 4
Geological Arrangement in Hunt's Example (1985)

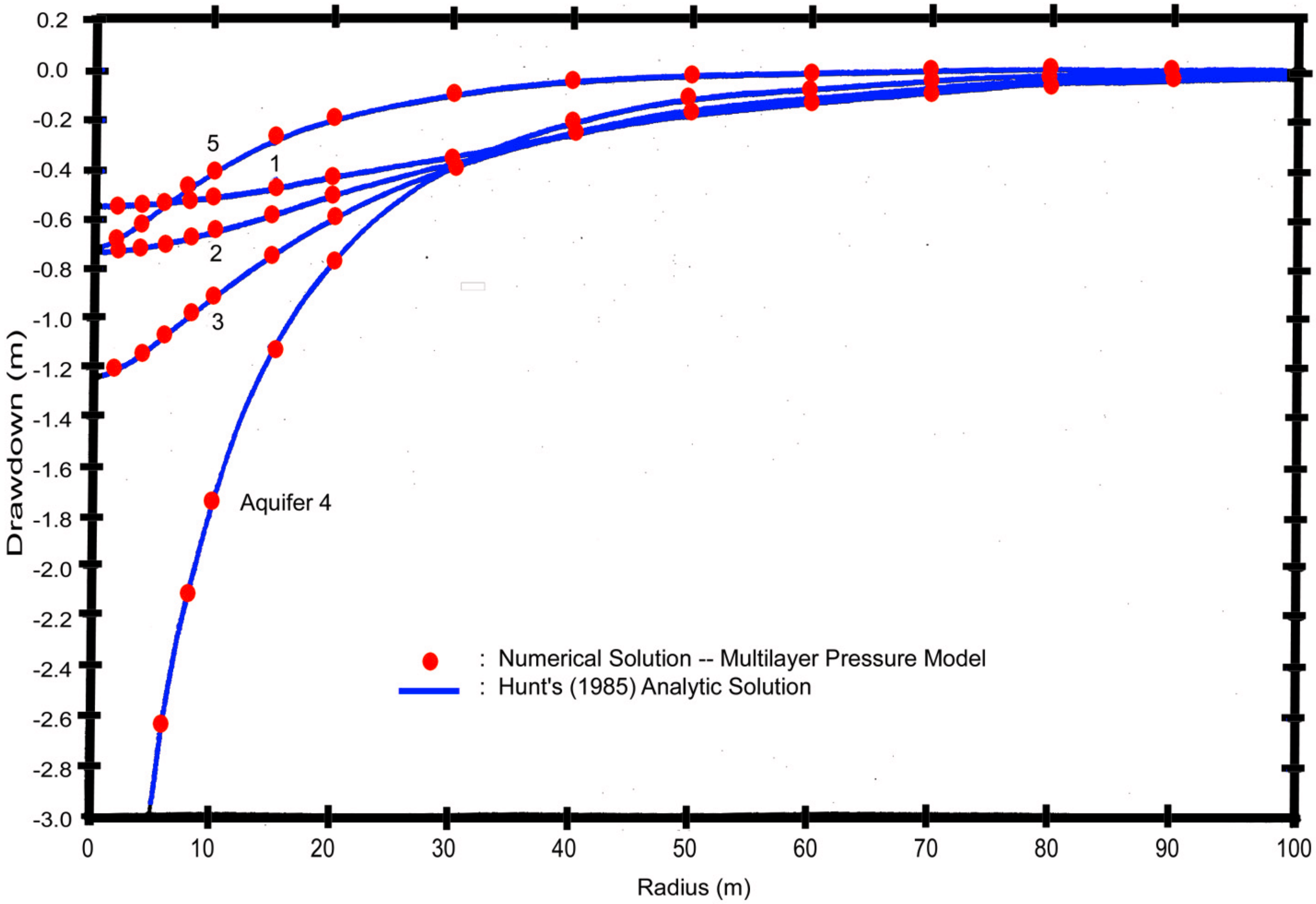


Figure 5: Steady State Drawdown Distribution in Each Aquifer.

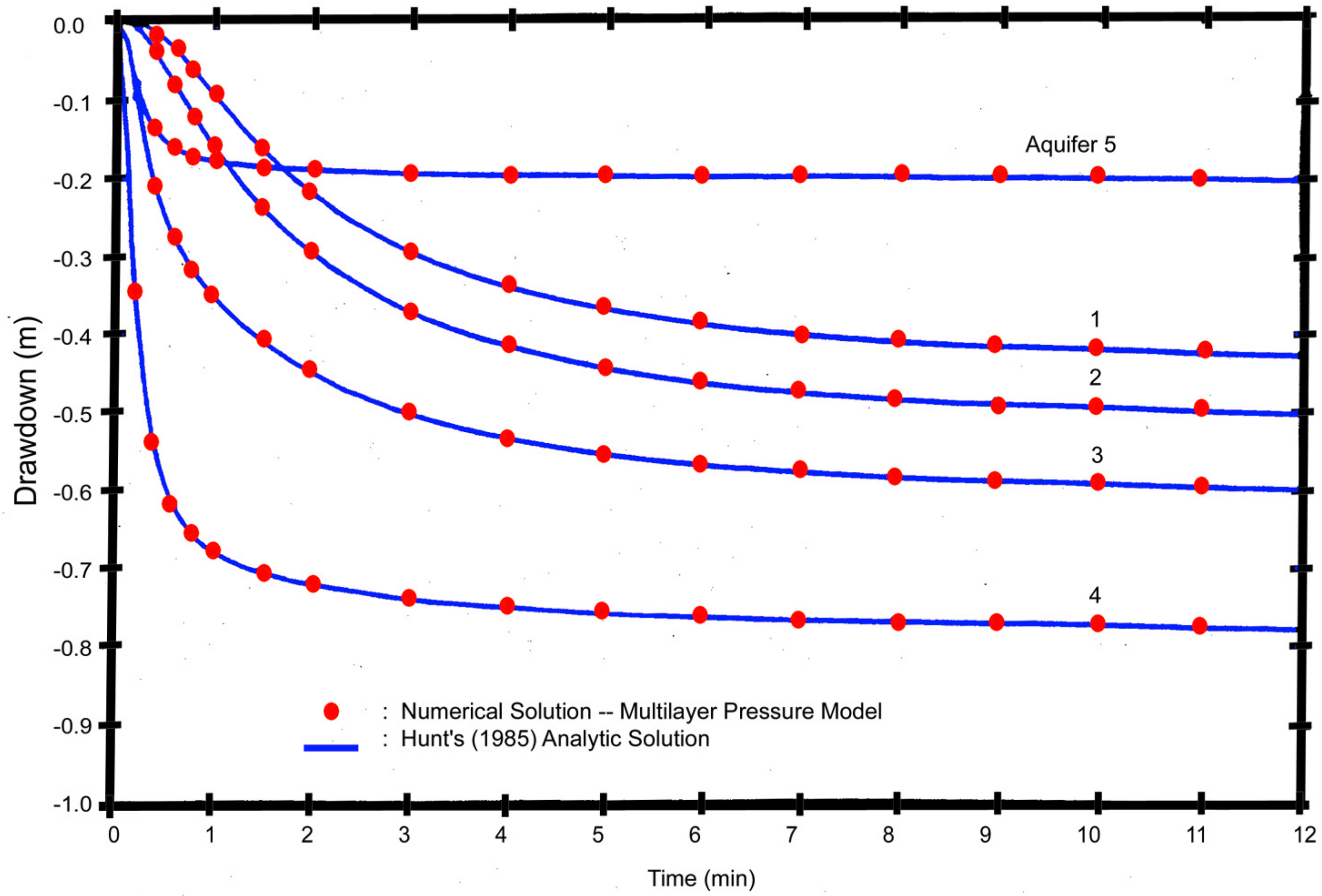


Figure 6: Time Dependent Drawdown Variations at r = 20m in Each Aquifer.